Research Paper I

Accuracy of Pawan Upadhyay's Pressure Curvature Law of Gravity

Claim A: PPC Tensor-Metric Equation, Vacuum Limit, Geodesic Motion, and Mercury's Perihelion Precession

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Proposed Theory: Pawan Upadhyay's Pressure Curvature Law of Gravity (PPC Law of Gravity)
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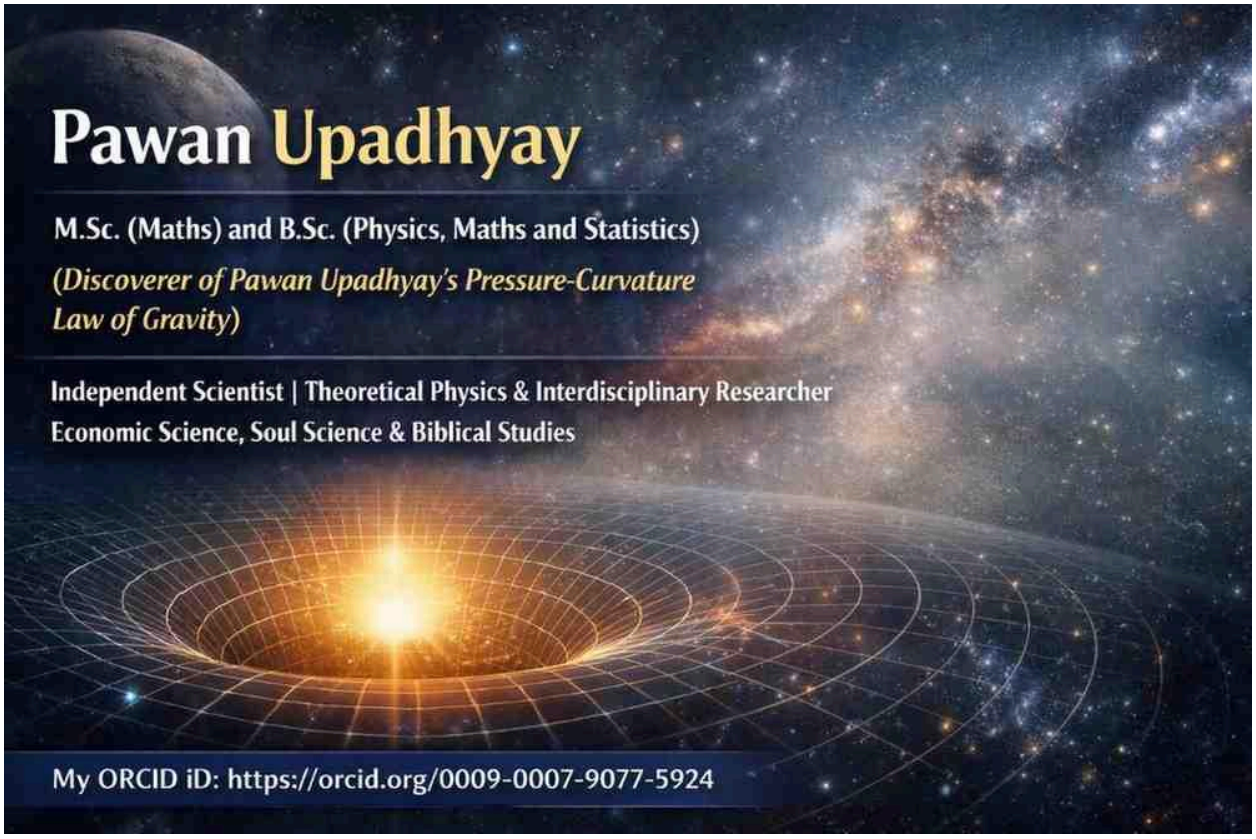
Pawan Upadhyay's

Pressure Curvature Law of Gravity

(PPC Law of Gravity)

by Mr. Pawan Upadhyay, Independent Scientist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Abstract

This paper develops and tests **Claim A** of Pawan Upadhyay's Pressure Curvature Law of Gravity (PPC). The central feature of the PPC formulation is that gravitational energy density E_d and gravitational pressure P_g determine the spacetime metric, while the metric determines the Christoffel symbols and therefore the geodesic motion of matter.

The fundamental PPC quantities are

$E_d = \rho c^2$

and

$P_g=\omega E_d$

The PPC geodesic motion equation is

$$\boxed{ \frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu_{\alpha\beta} \left(g_{\mu\nu}(E_d,P_g)\right) \frac{dx^\alpha}{d\tau} \frac{dx^\beta}{d\tau} =0. }$$

$d^2x^\mu/d\tau^2 + \Gamma^\mu_{\alpha\beta}(g_{\mu\nu}(E_d, P_g)) (dx^\alpha/d\tau)(dx^\beta/d\tau) = 0.$

$$\frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu_{\alpha\beta}\left(g_{\mu\nu}(E_d, P_g)\right)\frac{dx^\alpha}{d\tau}\frac{dx^\beta}{d\tau} = 0.$$

This equation expresses the central PPC hierarchy:

\[\boxed{ (E_d,P_g) \rightarrow g_{\mu\nu} \rightarrow \Gamma^\mu_{\alpha\beta} \rightarrow \text{geodesic motion}. } \]

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$$\boxed{(E_d, P_g) \rightarrow g_{\mu\nu} \rightarrow \Gamma^\mu_{\alpha\beta} \rightarrow \text{geodesic motion.}}$$

The PPC weak-field static equation is

$\nabla^2\Phi = (4\pi G/c^2)(E_d + 3P_g)$.

The purpose of Claim A is to investigate the vacuum exterior of the gravitating source. When the local exterior satisfies

$E_d=P_g=0$,

the PPC tensor field equation reduces to the vacuum curvature equation

$G_{\mu\nu} = 0$.

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Under the assumptions of static spherical symmetry and asymptotic flatness, the corresponding exterior metric is the Schwarzschild metric. The geodesic equation then produces the standard relativistic perihelion advance
\[\boxed{ \Delta\phi = \frac{6\pi GM_\odot} {a(1-e^2)c^2} } \]

and approximately

\[\boxed{ 42.98"/\text{century} \approx43"/\text{century}. } \]

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$$\boxed{\Delta\phi = \frac{6\pi G M_\odot}{a(1-e^2)c^2}}$$

and, for Mercury, approximately

$$\boxed{42.98''/\text{century} \approx 43''/\text{century}.}$$

Thus, **Claim A establishes that the PPC tensor-metric framework recovers the General-Relativistic vacuum prediction for Mercury.**

Thus Claim A establishes that the PPC tensor-metric framework **recovers the General-Relativistic vacuum prediction** for Mercury.

It does not, by itself, establish that the PPC pressure-gradient mechanism is an independent explanation of the (43") effect. That stronger question is treated separately as Claim B.

---

1. Introduction

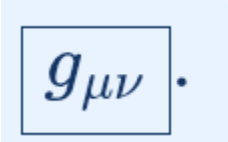
The PPC theory describes gravity through gravitational pressure, energy density, spacetime geometry, and motion.

The important conceptual point is that the theory should not be represented only by the scalar Newtonian-like relation

g = −∇Φ. (1)

Instead, the PPC theory is formulated on a spacetime metric

g_μv



The attached PPC formulation gives the geodesic equation in the form

\[\boxed{ \frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu_{\alpha\beta} \left(g_{\mu\nu}(E_d,P_g)\right) \frac{dx^\alpha}{d\tau} \frac{dx^\beta}{d\tau} =0. } \tag 2\]

$$\frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu_{\alpha\beta}\left(g_{\mu\nu}(E_d, P_g)\right)\frac{dx^\alpha}{d\tau}\frac{dx^\beta}{d\tau} = 0.$$

The accompanying interpretation is:

1. Christoffel symbols depend on the metric.
2. The metric depends on (E_d) and (P_g).
3. Therefore the geodesic motion is ultimately governed by the PPC gravitational fields.

This paper takes that equation as the starting point.

---

2. PPC Fundamental Variables

Let

$x^\mu = (ct, x, y, z)$.

The matter mass density is

$\rho = \rho(x, y, z, ct)$.

$E_d(x, y, z, ct) = \rho(x, y, z, ct)c^2$. (4)

The equation-of-state parameter is

$\omega = \omega(x, y, z, ct)$.

The PPC gravitational pressure is

$P_g(x, y, z, ct) = \omega(x, y, z, ct)E_d(x, y, z, ct)$. (5)

Therefore,

$P_g = \omega \rho c^2$. (6)

---

3. PPC Metric

The fundamental geometric object is

$g_{\mu\nu} = g_{\mu\nu}(E_d, P_g)$. (7)

This notation means that the metric is determined by the PPC gravitational source variables.

A static spherical metric may be written as

$ds^2 = -e^{2\nu(r)}c^2dt^2 + e^{2\lambda(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$. (8)

The functions

$\nu(r), \quad \lambda(r)$

are determined by the gravitational field equations and their boundary conditions.

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---

4. Christoffel Symbols in PPC Theory

The Christoffel symbols are

$$\boxed{ \Gamma^\mu_{\alpha\beta} = \frac12g^{\mu\nu} \left(\frac{\partial g_{\nu\beta}}{\partial x^\alpha} + \frac{\partial g_{\nu\alpha}}{\partial x^\beta} - \frac{\partial g_{\alpha\beta}}{\partial x^\nu} \right). } \tag{9}$$

Because

$$g_{\mu\nu}=g_{\mu\nu}(E_d,P_g),$$

the notation in the attached PPC equation,

$$\Gamma^\mu_{\alpha\beta} \left(g_{\mu\nu}(E_d,P_g)\right),$$

means that the connection is generated by the PPC metric.

More explicitly,

$$ \boxed{ \Gamma^\mu_{\alpha\beta} = \Gamma^\mu_{\alpha\beta} \left[g(E_d,P_g), \partial g(E_d,P_g) \right]. } \tag{10} $$

Thus, whenever E_d and P_g vary spatially or temporally, their variation can influence the metric and consequently the connection.

The Christoffel symbols are

$$\boxed{\Gamma^{\mu}_{\alpha\beta} = \frac{1}{2} g^{\mu\nu} \left(\frac{\partial g_{\nu\beta}}{\partial x^{\alpha}} + \frac{\partial g_{\nu\alpha}}{\partial x^{\beta}} - \frac{\partial g_{\alpha\beta}}{\partial x^{\nu}} \right).} \tag{9}$$

Because

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Thus, whenever E_d and P_g vary spatially or temporally, their variation can influence the metric and consequently the connection.

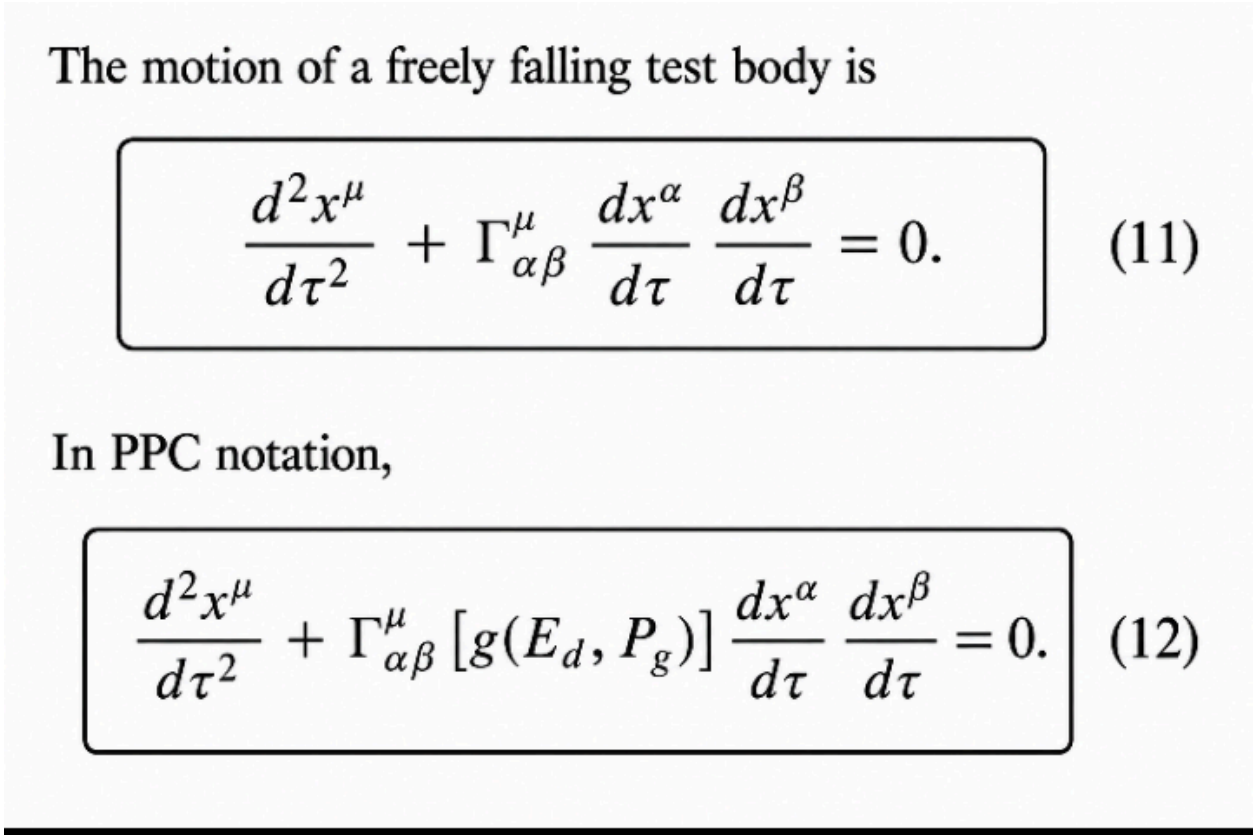
5. PPC Geodesic Motion

The motion of a freely falling test body is

$$\boxed{ \frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu_{\alpha\beta} \frac{dx^\alpha}{d\tau} \frac{dx^\beta}{d\tau} =0. } \tag{11}$$

In PPC notation,

$$\boxed{ \frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu_{\alpha\beta} [g(E_d,P_g)] \frac{dx^\alpha}{d\tau} \frac{dx^\beta}{d\tau} =0. } \tag{12}$$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This is the central motion equation of the PPC tensor-metric theory.

It replaces the idea that gravity is merely an ordinary force acting on a particle. The particle follows a geodesic of the PPC spacetime metric.

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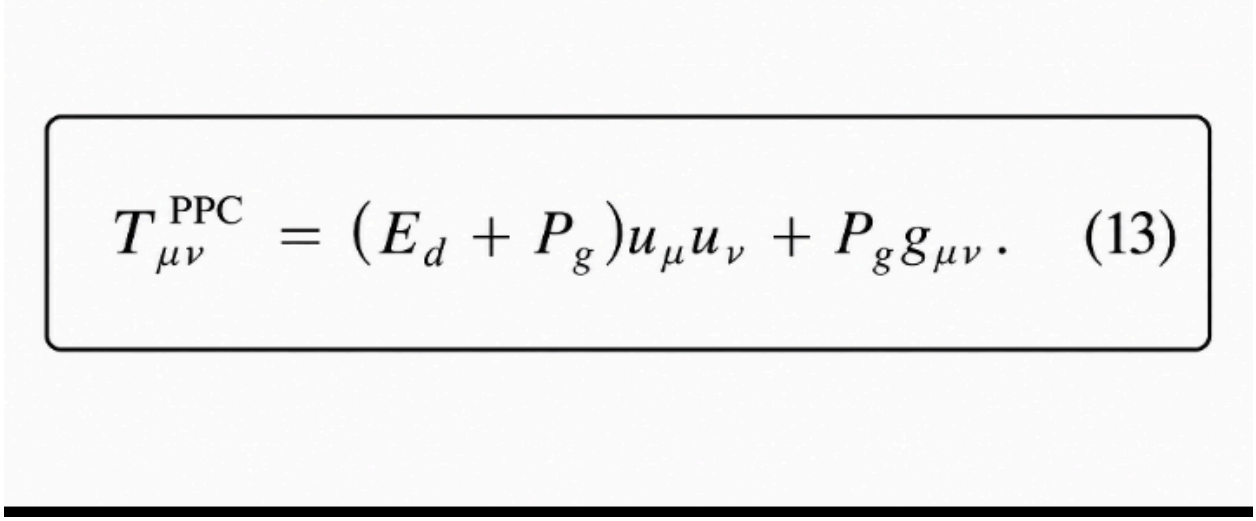
6. PPC Tensor Source

The PPC perfect-fluid energy-momentum tensor is

\[\boxed{ T_{\mu\nu}^{\rm PPC} = (E_d+P_g)u_\mu u_\nu + P_g g_{\mu\nu}. } \tag{13} \]



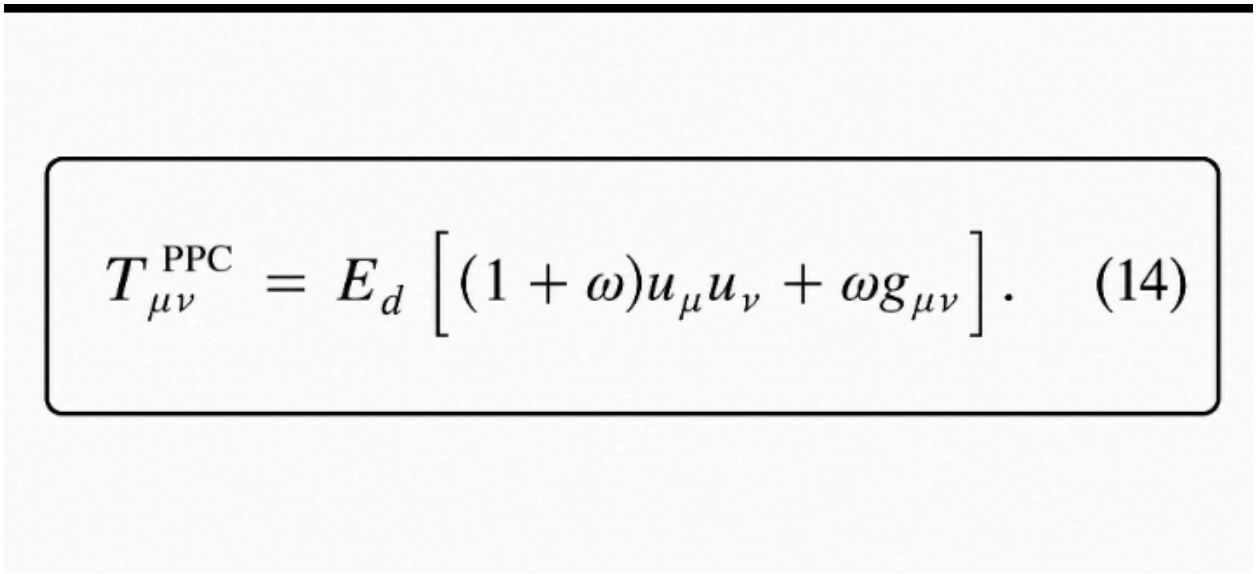
$$T_{\mu\nu}^{\mathrm{PPC}} = (E_d + P_g)u_\mu u_\nu + P_g g_{\mu\nu}. \quad (13)$$

Using

P_g=ω E_d

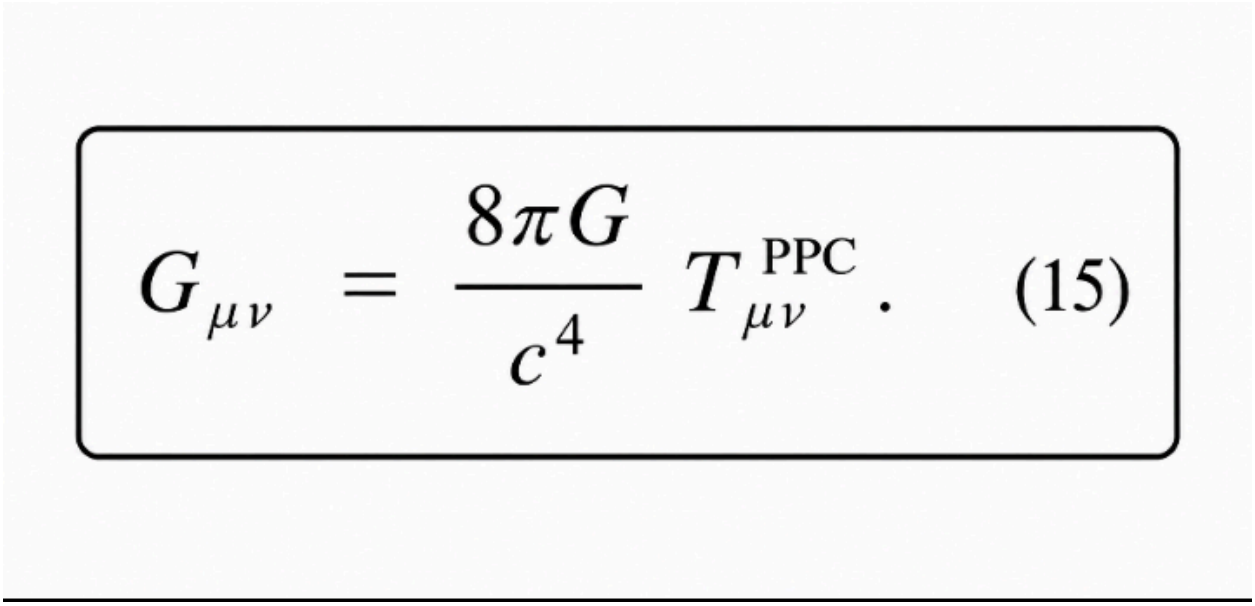
this becomes

\[\boxed{ T_{\mu\nu}^{\rm PPC} = E_d \left[(1+\omega)u_\mu u_\nu + \omega g_{\mu\nu} \right]. } \tag{14} \]



$$T_{\mu\nu}^{\mathrm{PPC}} = E_d \left[(1 + \omega)u_\mu u_\nu + \omega g_{\mu\nu} \right]. \quad (14)$$

The corresponding tensor field equation is taken as
\[\boxed{ G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}^{\rm PPC}. } \tag{15} \]



$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}^{\mathrm{PPC}} . \qquad (15)$$

The metric is consequently determined by the PPC energy and pressure source, subject to boundary conditions.

---

7. Weak-Field PPC Limit

In the weak-field limit,

$g_{00} \simeq -(1 + 2\Phi/c^2)$. (16)

The PPC source produces

$\nabla^2\Phi = (4\pi G/c^2)(E_d + 3P_g)$. (17)

Using

$E_d = \rho c^2, \quad P_g = \omega E_d,$

we obtain

$\nabla^2\Phi = 4\pi G\rho(1 + 3\omega)$. (18)

This connects the PPC tensor formulation to its static weak-field formulation.

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This connects the PPC tensor formulation to its static weak-field formulation.

8. Claim A: Solar Exterior

Mercury moves predominantly through the solar exterior.

For the **strict local vacuum model**,

$$\boxed{ \rho=0. }$$

Therefore,

$$\boxed{ E_d=0. }$$

With

$$P_g=\omega E_d,$$

we obtain

$$\boxed{ P_g=0. }$$

The PPC energy-momentum tensor becomes

$$\boxed{ T_{\mu\nu}^{\rm PPC}=0. }$$

Therefore,

$$\boxed{ G_{\mu\nu}=0. } \tag{19}$$

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Therefore,

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This is the PPC vacuum limit.

9. Vacuum Metric

For a static, spherically symmetric, asymptotically flat vacuum, the resulting metric is

\[\boxed{ ds^2 = -\left(1-\frac{2GM_\odot}{c^2r}\right)c^2dt^2 + \left(1-\frac{2GM_\odot}{c^2r}\right)^{-1}dr^2 + r^2d\Omega^2. } \tag{20} \]

The corresponding Christoffel symbols are calculated from this metric.

The PPC geodesic equation therefore becomes the geodesic equation of the Schwarzschild exterior.

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The corresponding Christoffel symbols are calculated from this metric.

The PPC geodesic equation therefore becomes the geodesic equation of the Schwarzschild exterior.

10. Mercury's Geodesic Orbit

For equatorial motion,

$$ \theta=\frac{\pi}{2}, $$

and defining

$$ u=\frac1r, $$

the Schwarzschild geodesic equation gives, to first post-Newtonian order,

$$ \boxed{ \frac{d^2u}{d\phi^2}+u = \frac{GM_\odot}{h^2} + \frac{3GM_\odot}{c^2}u^2. } \tag{21} $$

The additional term

$$ \boxed{ \frac{3GM_\odot}{c^2}u^2 } $$

causes the orbit to precess.

The corresponding perihelion advance is

$$ \boxed{ \Delta\phi = \frac{6\pi GM_\odot} {a(1-e^2)c^2}. } \tag{22} $$

For Mercury,

$$ \boxed{ \Delta\phi \approx42.98''/\text{century}. } \tag{23} $$

Rounded,

$$ \boxed{ \Delta\phi\approx43''/\text{century}. } $$

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Rounded,

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11. Result of Claim A

The complete chain is

$$\boxed{ (E_d,P_g) \rightarrow g_{\mu\nu}^{\rm PPC} \rightarrow \Gamma^\mu_{\alpha\beta} \rightarrow \text{geodesic motion}. }$$

For the local vacuum,

$$\boxed{ E_d=P_g=0 }$$

gives

$$\boxed{ G_{\mu\nu}=0, }$$

which gives the Schwarzschild exterior and therefore

$$\boxed{ \Delta\phi\simeq42.98''/\text{century}. }$$

Claim A conclusion

$$\boxed{ \textbf{PPC recovers the General-Relativistic vacuum prediction for Mercury's perihelion precession.} }$$

This is a **recovery/compatibility result**.

It is not yet an independent experimental distinction between PPC and GR.

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Claim A conclusion

PPC recovers the General-Relativistic vacuum prediction for Mercury's perihelion precession.

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Research Paper II

Accuracy of Pawan Upadhyay's Pressure Curvature Law of Gravity

Claim B: Non-Zero Gravitational Pressure Gradient, PPC Metric, Geodesic Motion, and Mercury's Perihelion Precession

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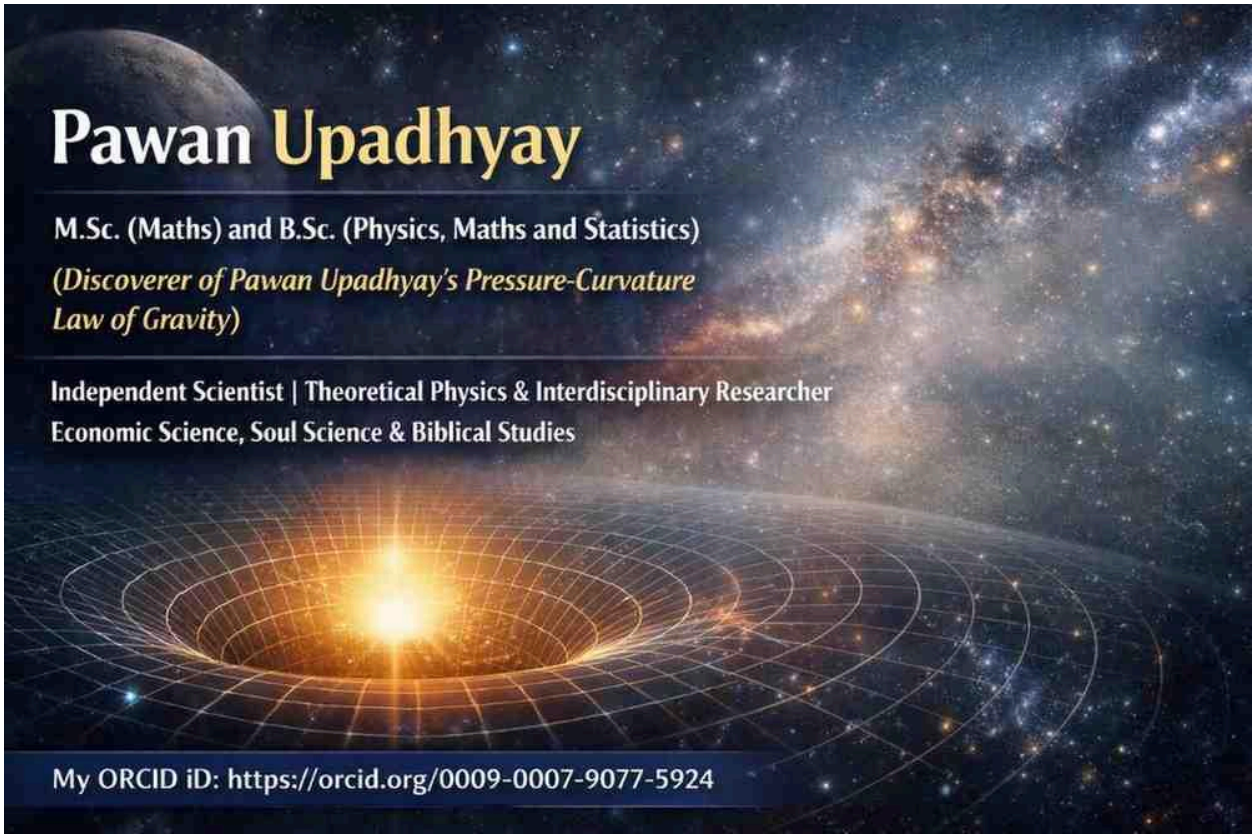
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Abstract

This paper develops **Claim B**, the PPC-specific pressure-curvature test.

Unlike Claim A, which considers the strict vacuum limit $E_d=P_g=0$, Claim B investigates the case

$$\boxed{ \nabla P_g\neq0. }$$

The fundamental PPC relations are

$$\boxed{ E_d=\rho c^2 }$$

and

$$\boxed{ P_g=\omega E_d. }$$

The PPC pressure-force relation is

$$\boxed{ \mathbf F=\rho\mathbf g=-\nabla P_g. } \tag{24}$$

The pressure field is incorporated into the spacetime metric,

$$\boxed{ g_{\mu\nu}=g_{\mu\nu}(E_d,P_g), }$$

and the resulting Christoffel symbols are

$$\boxed{ \Gamma^\mu_{\alpha\beta} = \Gamma^\mu_{\alpha\beta} [g(E_d,P_g)]. }$$

The motion of a test body is consequently governed by

$$\boxed{ \frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu_{\alpha\beta} [g(E_d,P_g)] \frac{dx^\alpha}{d\tau} \frac{dx^\beta}{d\tau} =0. } \tag{25}$$

This paper develops Claim B, the PPC-specific pressure-curvature test.

Unlike Claim A, which considers the strict vacuum limit $E_d = P_g = 0$, Claim B investigates the case

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The central question of Claim B is whether a non-zero PPC pressure gradient can generate the post-Newtonian orbital correction required for Mercury's approximately (43'') per century perihelion advance.

A successful Claim B calculation must derive the correction from the specified PPC fields rather than inserting the Schwarzschild result by assumption.

---

1. Claim B Hypothesis

The PPC hypothesis can be expressed as

$P_g = \omega E_d$

together with

$\nabla P_g \neq 0.$

Therefore,

$\nabla P_g = \nabla(\omega E_d).$ (26)

Since

$E_d = \rho c^2,$

we have

$\nabla P_g = c^2 \nabla(\omega \rho).$ (27)

Thus the PPC pressure gradient can arise from either

$\nabla \rho \neq 0,$

or

$\nabla \omega \neq 0,$

or both.

---

2. PPC Pressure Force

The PPC force density is

$F = -\nabla P_g$. (28)

Using

$P_g = \omega\rho c^2$,

we obtain

$F = -c^2\nabla(\omega\rho)$. (29)

The gravitational force density is simultaneously

$F = \rho g$. (30)

Therefore,

$\rho g = -\nabla P_g$. (31)

This is the fundamental PPC pressure-gradient relation to be tested.

---

3. Covariant PPC Pressure Relation

In a tensor formulation, the ordinary spatial gradient must be represented covariantly.

Define

$$\boxed{ h_{\mu\nu}=g_{\mu\nu}+u_\mu u_\nu. }$$

The spatial pressure gradient is

$$\boxed{ h_\mu{}^\nu\nabla_\nu P_g. }$$

The corresponding fluid-force relation can be expressed as

$$\boxed{ (E_d+P_g)a_\mu = -h_\mu{}^\nu\nabla_\nu P_g. } \tag{32}$$

Thus the PPC pressure force is incorporated directly into the relativistic geometry.

In the weak-field limit, this reduces to the intended PPC form

\[\boxed{ \mathbf F=-\nabla P_g. } \]

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h_μv = g_μv + u_μ u_v.

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F = −∇P_g.

Note: The definition h_μv = g_μv + u_μ u_v assumes the four-velocity is normalized as u_μ u_μ = −1. If u_μ is defined with physical units so that u_μ u_μ = −c², the projector should instead be hμv = g_μv + u_μ u_v/c².

TENSOR FORMULATION OF THE PPC PRESSURE GRADIENT

In a tensor formulation, the ordinary spatial gradient must be represented covariantly.

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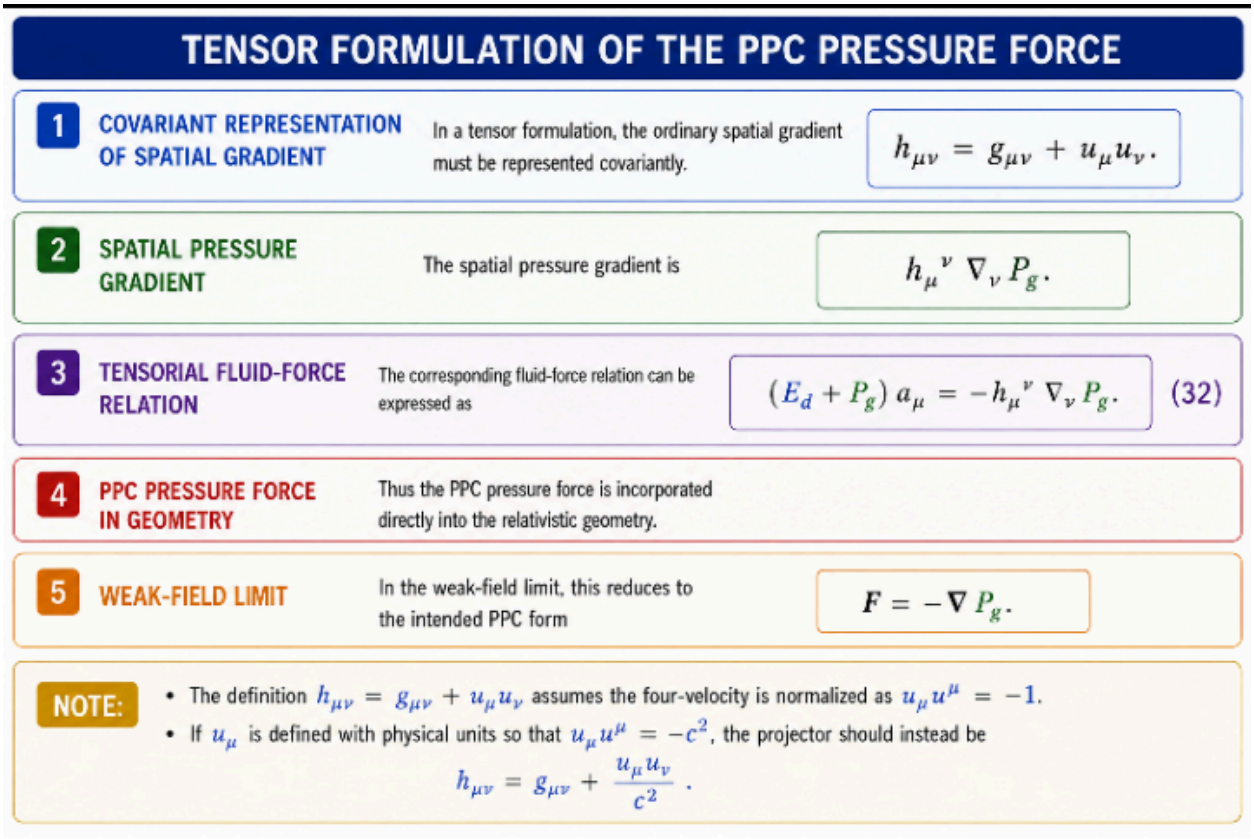
TENSOR FORMULATION OF THE PPC PRESSURE FORCE
1
COVARIANT REPRESENTATION OF SPATIAL GRADIENT
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$h_{\mu\nu} = g_{\mu\nu} + u_\mu u_\nu .$
2
SPATIAL PRESSURE GRADIENT
The spatial pressure gradient is
$h_\mu{}^\nu \nabla_\nu P_g .$
3
TENSORIAL FLUID-FORCE RELATION
The corresponding fluid-force relation can be expressed as
$(E_d + P_g)\, a_\mu = -h_\mu{}^\nu \nabla_\nu P_g .$
(32)
4
PPC PRESSURE FORCE IN GEOMETRY
Thus the PPC pressure force is incorporated directly into the relativistic geometry.
5
WEAK-FIELD LIMIT
In the weak-field limit, this reduces to the intended PPC form
$\boldsymbol{F} = -\nabla P_g .$
NOTE:
• The definition $h_{\mu\nu} = g_{\mu\nu} + u_\mu u_\nu$ assumes the four-velocity is normalized as $u_\mu u^\mu = -1$.
• If u_μ is defined with physical units so that $u_\mu u^\mu = -c^2$, the projector should instead be
$h_{\mu\nu} = g_{\mu\nu} + \frac{u_\mu u_\nu}{c^2} .$

---

4. PPC Metric Dependence

The attached PPC geodesic equation gives the essential relation

\[\boxed{ \Gamma^\mu_{\alpha\beta} \left(g_{\mu\nu}(E_d,P_g)\right). } \tag{33} \]

Γ^μ_αβ (g_μν(E_d, P_g)). (33)

This should be interpreted as the following hierarchy:

\[\boxed{ E_d,P_g \rightarrow g_{\mu\nu} \rightarrow \Gamma^\mu_{\alpha\beta}. } \]

E_d, P_g → g_μν → Γ^μ_αβ.

Because

\[\Gamma^\mu_{\alpha\beta} = \frac12 g^{\mu\nu} (\partial_\alpha g_{\nu\beta} + \partial_\beta g_{\nu\alpha} - \partial_\nu g_{\alpha\beta}), \]

Γ^μ_αβ = ½ g^μν (∂_α g_νβ + ∂_β g_να − ∂_ν g_αβ).

any spatial or temporal variation of the PPC fields that changes the metric also changes the connection.

More explicitly,

$$\boxed{ g_{\mu\nu} = g_{\mu\nu}[E_d,P_g] }$$

g_μv = g_μv[E_d, P_g]

implies

$$\boxed{ \Gamma^\mu_{\alpha\beta} = \Gamma^\mu_{\alpha\beta} [E_d,P_g,\partial E_d,\partial P_g,\ldots]. } \tag{34}$$

Γ^μ_αβ = Γ^μ_αβ [E_d, P_g, ∂E_d, ∂P_g, …]. (34)

This is the central mathematical idea of Claim B.

CLAIM B — CENTRAL MATHEMATICAL IDEA

PPC GEODESIC EQUATION GIVES THE ESSENTIAL RELATION

$$\Gamma^{\mu}_{\alpha\beta}\left(g_{\mu\nu}(E_d, P_g)\right). \qquad (33)$$

HIERARCHY

$$E_d, P_g \rightarrow g_{\mu\nu} \rightarrow \Gamma^{\mu}_{\alpha\beta}$$

BECAUSE THE CHRISTOFFEL SYMBOL IS DETERMINED BY THE METRIC AND ITS DERIVATIVES,

$$\Gamma^{\mu}_{\alpha\beta} = \frac{1}{2} g^{\mu\nu}\left(\partial_\alpha g_{\nu\beta} + \partial_\beta g_{\nu\alpha} - \partial_\nu g_{\alpha\beta}\right),$$

any spatial or temporal variation of the PPC fields that changes the metric also changes the connection.

METRIC FROM PPC FIELDS

$$g_{\mu\nu} = g_{\mu\nu}\left[E_d,\, P_g\right]$$

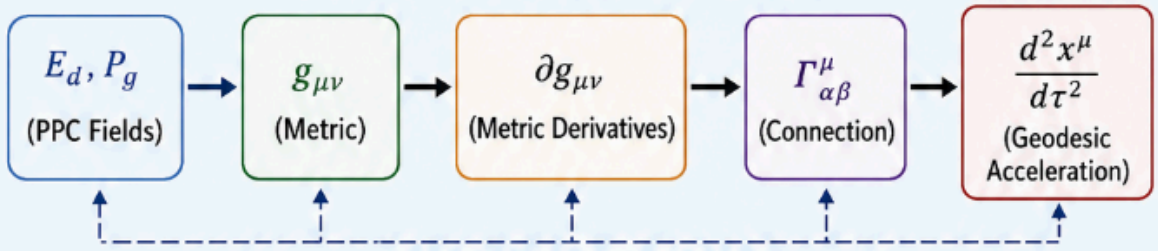
IMPLIES →

CONNECTION DEPENDENCE

$$\Gamma^{\mu}_{\alpha\beta} = \Gamma^{\mu}_{\alpha\beta}\left[E_d, P_g, \partial E_d, \partial P_g, \ldots\right] \qquad (34)$$

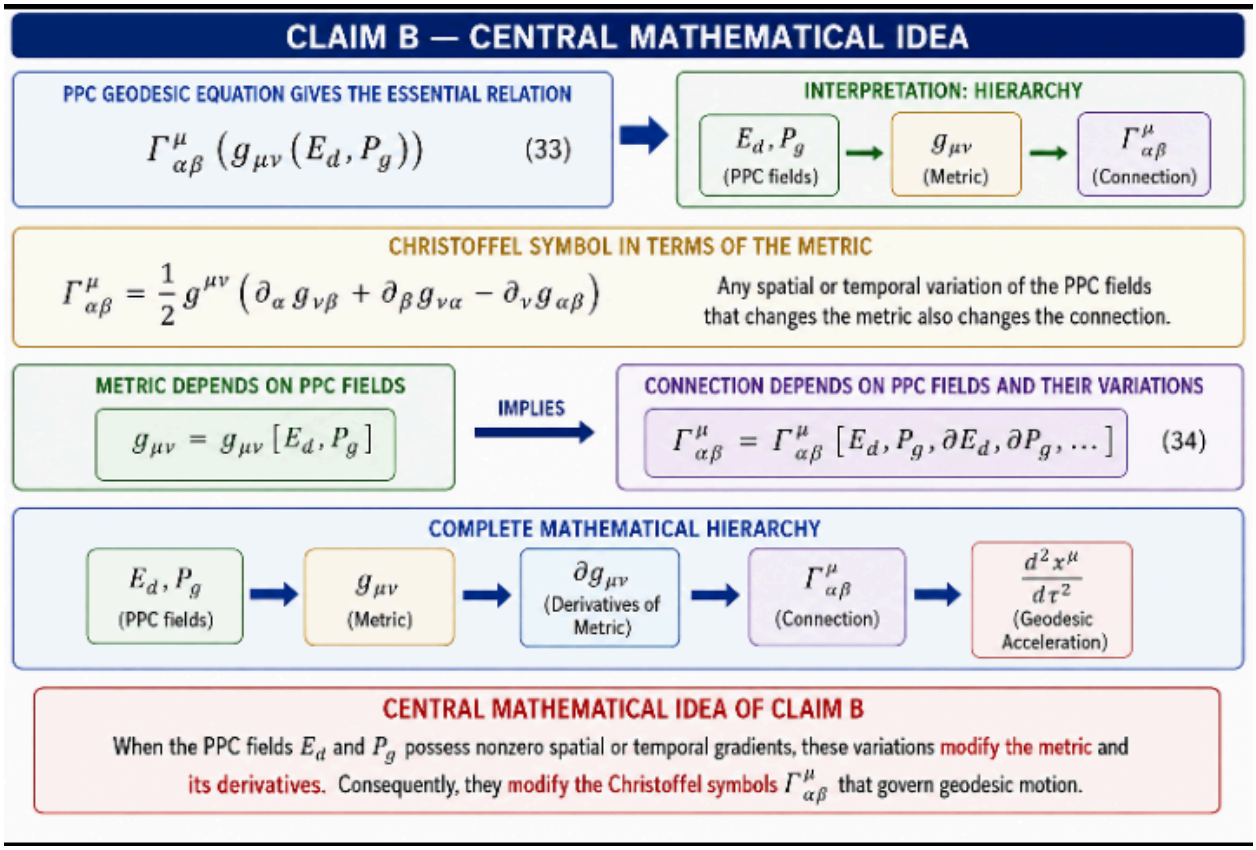
Thus, the PPC fields E_d and P_g determine the metric, while their spatial and temporal variations enter through the derivatives of the metric and consequently modify the Christoffel symbols.

COMPLETE MATHEMATICAL HIERARCHY



CENTRAL MATHEMATICAL IDEA OF CLAIM B

When the PPC fields E_d and P_g possess nonzero spatial or temporal gradients, these variations **modify the metric and its derivatives**. Consequently, they **modify the Christoffel symbols** $\Gamma^{\mu}_{\alpha\beta}$ that govern geodesic motion.


CLAIM B — CENTRAL MATHEMATICAL IDEA
PPC GEODESIC EQUATION GIVES THE ESSENTIAL RELATION
$\Gamma^{\mu}_{\alpha\beta}\left(g_{\mu\nu}\left(E_d, P_g\right)\right)$ (33)
INTERPRETATION: HIERARCHY
E_d, P_g (PPC fields)
$g_{\mu\nu}$ (Metric)
$\Gamma^{\mu}_{\alpha\beta}$ (Connection)
CHRISTOFFEL SYMBOL IN TERMS OF THE METRIC
$\Gamma^{\mu}_{\alpha\beta} = \frac{1}{2} g^{\mu\nu}\left(\partial_\alpha g_{\nu\beta} + \partial_\beta g_{\nu\alpha} - \partial_\nu g_{\alpha\beta}\right)$
Any spatial or temporal variation of the PPC fields that changes the metric also changes the connection.
METRIC DEPENDS ON PPC FIELDS
$g_{\mu\nu} = g_{\mu\nu}[E_d, P_g]$
IMPLIES
CONNECTION DEPENDS ON PPC FIELDS AND THEIR VARIATIONS
$\Gamma^{\mu}_{\alpha\beta} = \Gamma^{\mu}_{\alpha\beta}[E_d, P_g, \partial E_d, \partial P_g, \ldots]$ (34)
COMPLETE MATHEMATICAL HIERARCHY
E_d, P_g (PPC fields)
$g_{\mu\nu}$ (Metric)
$\partial g_{\mu\nu}$ (Derivatives of Metric)
$\Gamma^{\mu}_{\alpha\beta}$ (Connection)
$\frac{d^2 x^\mu}{d\tau^2}$ (Geodesic Acceleration)
CENTRAL MATHEMATICAL IDEA OF CLAIM B
When the PPC fields E_d and P_g possess nonzero spatial or temporal gradients, these variations modify the metric and its derivatives. Consequently, they modify the Christoffel symbols $\Gamma^{\mu}_{\alpha\beta}$ that govern geodesic motion.

---

5. PPC Tensor Equation

The PPC perfect-fluid tensor is

\[\boxed{ T_{\mu\nu}^{\rm PPC} = (E_d+P_g)u_\mu u_\nu + P_g g_{\mu\nu}. } \tag{35} \]

PPC EFFECTIVE STRESS–ENERGY TENSOR

$$T_{\mu\nu}^{\mathrm{PPC}} = \left(E_d + P_g\right) u_\mu u_\nu + P_g\, g_{\mu\nu}. \qquad (35)$$

Where:

- E_d = effective energy density of the PPC field
- P_g = effective pressure (gravitational pressure) of the PPC field
- u_μ = 4-velocity of the fluid element $\left(u_\mu u^\mu = -1\right)$
- $g_{\mu\nu}$ = spacetime metric tensor

The tensor field equation is

$$\boxed{ G_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}^{\rm PPC}. } \tag{36}$$

PPC EINSTEIN FIELD EQUATION

$$G_{\mu\nu} = \frac{8\pi G}{c^4}\, T_{\mu\nu}^{\mathrm{PPC}}. \qquad (36)$$

Where:

- $G_{\mu\nu}$ = Einstein tensor (geometry of spacetime)
- $T_{\mu\nu}^{\mathrm{PPC}}$ = PPC effective stress–energy tensor
- G = Newton's gravitational constant
- c = Speed of light in vacuum

Consequently,

$$\boxed{ (E_d,P_g) \rightarrow T_{\mu\nu} \rightarrow G_{\mu\nu} \rightarrow g_{\mu\nu}. } \tag{37}$$

PPC Field-to-Metric Hierarchy

(E_d, P_g) → T_μv → G_μv → g_μv (37)

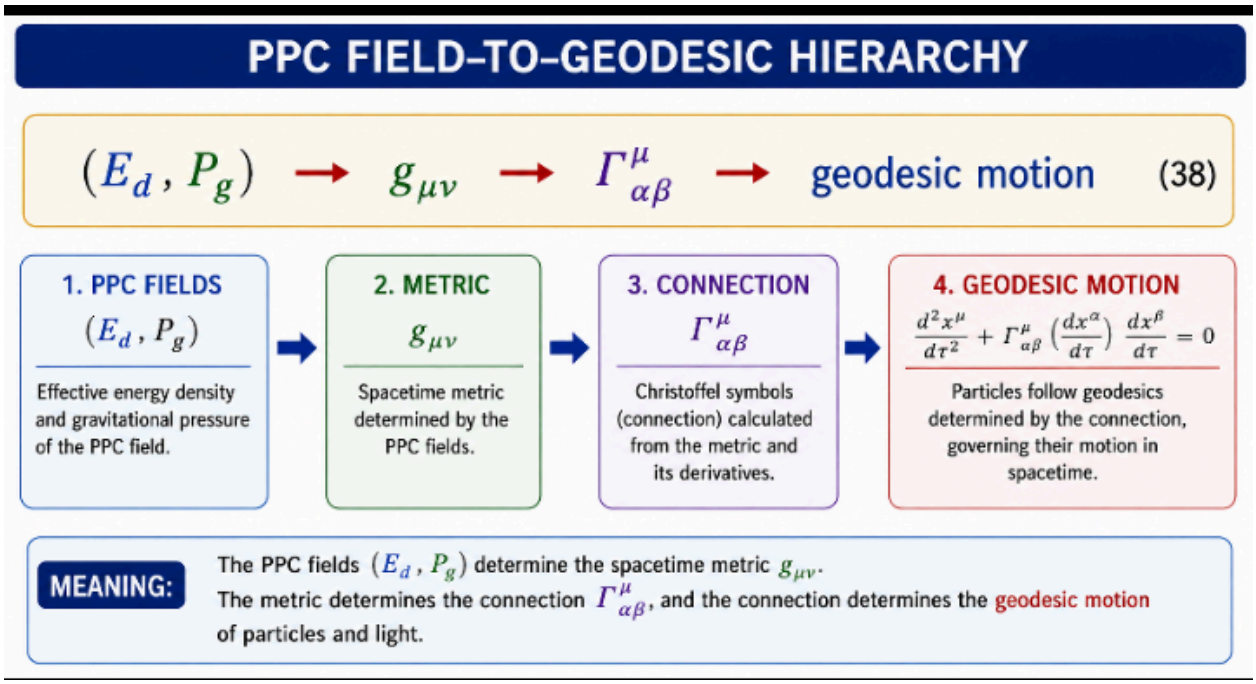
This represents the fundamental field-to-geometry hierarchy in the PPC framework:

PPC fields → Stress–energy tensor → Einstein tensor → Spacetime metric.

Combining this with the geodesic equation gives

\[\boxed{ (E_d,P_g) \rightarrow g_{\mu\nu} \rightarrow \Gamma^\mu_{\alpha\beta} \rightarrow \text{geodesic motion}. } \tag{38} \]

(E_d, P_g) → g_μv → Γ^μ(αβ) → geodesic motion. (38)


PPC FIELD–TO–GEODESIC HIERARCHY
(E_d, P_g) → $g_{\mu\nu}$ → $\Gamma^{\mu}_{\alpha\beta}$ → geodesic motion (38)
1. PPC FIELDS
(E_d, P_g)
Effective energy density and gravitational pressure of the PPC field.
2. METRIC
$g_{\mu\nu}$
Spacetime metric determined by the PPC fields.
3. CONNECTION
$\Gamma^{\mu}_{\alpha\beta}$
Christoffel symbols (connection) calculated from the metric and its derivatives.
4. GEODESIC MOTION
$\frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu_{\alpha\beta}\left(\frac{dx^\alpha}{d\tau}\right)\frac{dx^\beta}{d\tau} = 0$
Particles follow geodesics determined by the connection, governing their motion in spacetime.
MEANING:
The PPC fields (E_d, P_g) determine the spacetime metric $g_{\mu\nu}$.
The metric determines the connection $\Gamma^{\mu}_{\alpha\beta}$, and the connection determines the geodesic motion of particles and light.

“Light follows null geodesics, so its trajectory is determined by the curvature of spacetime encoded in the metric and its connection.”

For the PPC hierarchy:

PPC fields → metric → connection → null geodesic → light trajectory

In this formulation, light is not being treated as a massive particle experiencing an ordinary force. Its path is a **null geodesic of the PPC spacetime metric**.

---

6. Weak-Field PPC Curvature

The PPC weak-field equation is

$$\boxed{ \nabla^2\Phi = \frac{4\pi G}{c^2}(E_d+3P_g). } \tag{39}$$

Using

$$P_g=\omega E_d,$$

we obtain

$$\boxed{ \nabla^2\Phi = \frac{4\pi G}{c^2} E_d(1+3\omega). } \tag{40}$$

Since

$$E_d=\rho c^2,$$ $$\boxed{ \nabla^2\Phi = 4\pi G\rho(1+3\omega). } \tag{41}$$

Thus P_g enters the PPC curvature source itself, while

$$\nabla P_g$$

enters the pressure-force relation.

This gives two distinct PPC roles:

$$\boxed{ P_g \rightarrow \text{curvature source} }$$

and

$$\boxed{ \nabla P_g \rightarrow \text{pressure force}. }$$

PPC Weak-Field Equation and the Two Roles of Gravitational Pressure

The PPC weak-field equation is:

$\nabla^2\Phi = (4\pi G/c^2)(E_d + 3P_g)$ (39)

Using:

$P_g = \omega E_d$

we obtain:

$\nabla^2\Phi = (4\pi G/c^2)\, E_d(1 + 3\omega)$ (40)

Since:

$E_d = \rho c^2$

therefore:

$\nabla^2\Phi = 4\pi G\rho(1 + 3\omega)$ (41)

Thus, the gravitational pressure P_g enters the **PPC curvature source itself**, while its spatial gradient, ∇**P_g**, enters the pressure-force relation.

This gives two distinct PPC roles:

P_g → curvature source

and

∇P_g → pressure force

Therefore, **P_g and ∇P_g represent mathematically distinct contributions** in the PPC framework: P_g modifies the source responsible for spacetime curvature, whereas ∇P_g contributes to the local pressure-force dynamics.

---

7. Metric and Mercury's Motion

For a static spherical PPC geometry,

$$\boxed{ ds^2 = -e^{2\nu(r)}c^2dt^2 + e^{2\lambda(r)}dr^2 + r^2d\Omega^2. } \tag{42}$$

The metric functions must be determined from the PPC field equations with

$$E_d(r)$$

and

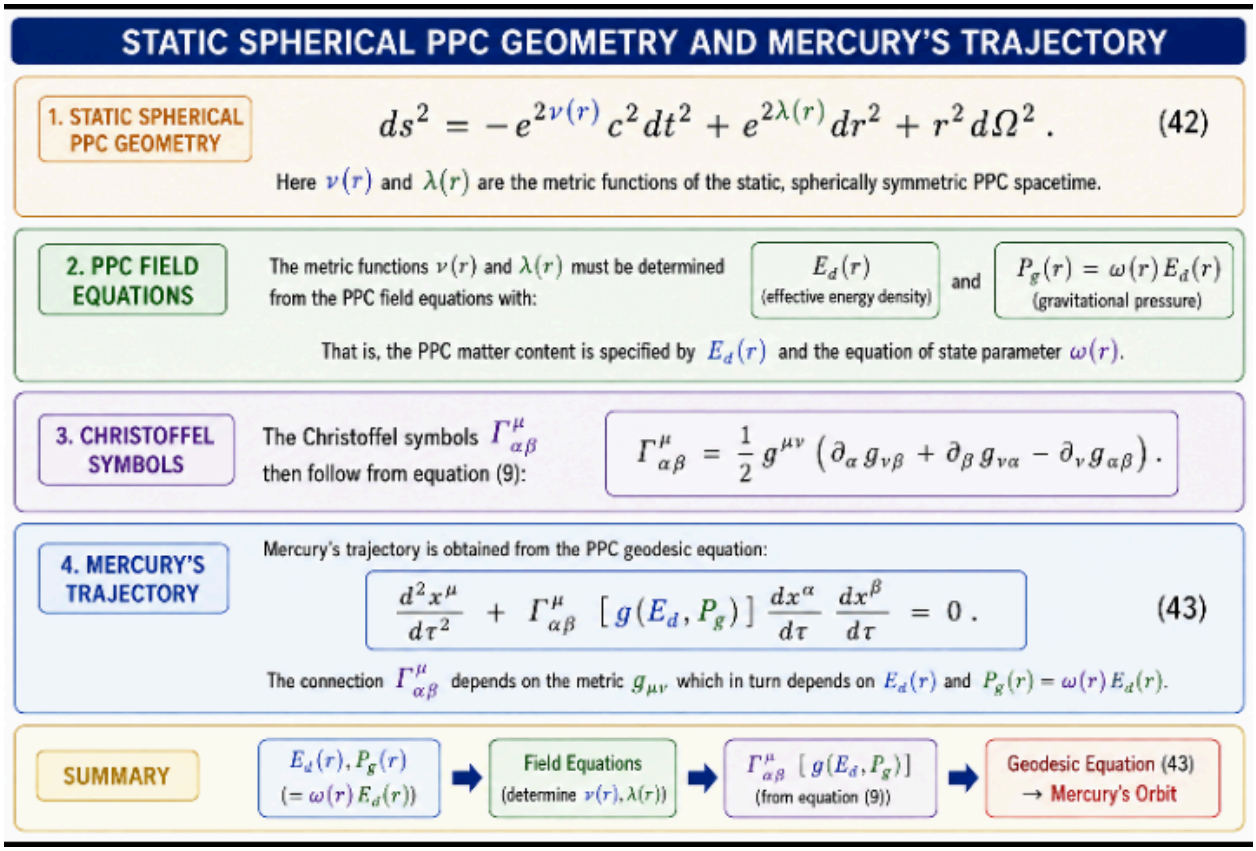
$$P_g(r)=\omega(r)E_d(r).$$

The Christoffel symbols then follow from equation (9).

Mercury's trajectory is obtained from

$$\boxed{ \frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu_{\alpha\beta} [g(E_d,P_g)] \frac{dx^\alpha}{d\tau} \frac{dx^\beta}{d\tau} =0. } \tag{43}$$

Thus the perihelion advance is not inserted separately; it must emerge from the PPC metric.


STATIC SPHERICAL PPC GEOMETRY AND MERCURY'S TRAJECTORY
1. STATIC SPHERICAL PPC GEOMETRY
$ds^2 = -e^{2\nu(r)}c^2dt^2 + e^{2\lambda(r)}dr^2 + r^2d\Omega^2$. (42)
Here $\nu(r)$ and $\lambda(r)$ are the metric functions of the static, spherically symmetric PPC spacetime.
2. PPC FIELD EQUATIONS
The metric functions $\nu(r)$ and $\lambda(r)$ must be determined from the PPC field equations with:
$E_d(r)$ (effective energy density)
and
$P_g(r) = \omega(r)E_d(r)$ (gravitational pressure)
That is, the PPC matter content is specified by $E_d(r)$ and the equation of state parameter $\omega(r)$.
3. CHRISTOFFEL SYMBOLS
The Christoffel symbols $\Gamma^\mu_{\alpha\beta}$ then follow from equation (9):
$\Gamma^\mu_{\alpha\beta} = \frac{1}{2}g^{\mu\nu}\left(\partial_\alpha g_{\nu\beta} + \partial_\beta g_{\nu\alpha} - \partial_\nu g_{\alpha\beta}\right)$.
4. MERCURY'S TRAJECTORY
Mercury's trajectory is obtained from the PPC geodesic equation:
$\frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu_{\alpha\beta}[g(E_d, P_g)]\frac{dx^\alpha}{d\tau}\frac{dx^\beta}{d\tau} = 0$. (43)
The connection $\Gamma^\mu_{\alpha\beta}$ depends on the metric $g_{\mu\nu}$ which in turn depends on $E_d(r)$ and $P_g(r) = \omega(r)E_d(r)$.
SUMMARY
$E_d(r), P_g(r)$ $(= \omega(r)E_d(r))$
Field Equations (determine $\nu(r), \lambda(r)$)
$\Gamma^\mu_{\alpha\beta}[g(E_d, P_g)]$ (from equation (9))
Geodesic Equation (43) → Mercury's Orbit

8. Required Mercury Test

Let

\[u=\frac1r. \]

The orbital equation obtained from the PPC metric must have the form

\[\boxed{ u''+u = \frac{GM_\odot}{h^2} + \delta_{\rm PPC}(u). } \tag{44} \]

For agreement with the observed relativistic correction, PPC must yield, at the relevant post-Newtonian order,

\[\boxed{ \delta_{\rm PPC}(u) = \frac{3GM_\odot}{c^2}u^2 } \tag{45} \]

or an mathematically equivalent correction that gives

\[\boxed{ \Delta\phi_{\rm PPC} \simeq42.98''/\text{century}. } \tag{46} \]

This is the decisive quantitative test of Claim B.

Let

$$\boxed{u = \frac{1}{r}\,.}$$

The orbital equation obtained from the PPC metric must have the form

$$\boxed{u'' + u = \frac{GM_\odot}{h^2} + \delta_{\rm PPC}(u)\,.} \tag{44}$$

For agreement with the observed relativistic correction, PPC must yield, at the relevant post-Newtonian order,

$$\boxed{\delta_{\rm PPC}(u) = \frac{3GM_\odot}{c^2}\,u^2} \tag{45}$$

or an mathematically equivalent correction that gives

$$\boxed{\Delta\phi_{\rm PPC} \simeq 42.98''/\text{century}\,.} \tag{46}$$

This is the decisive quantitative test of **Claim B**.

---

9. Why Claim B Is Different from Claim A

Claim A uses

$$\boxed{ E_d=P_g=0 }$$

in the exterior and therefore

$$\boxed{ G_{\mu\nu}=0. }$$

The resulting Schwarzschild metric automatically gives

$$42.98''/\text{century}.$$

Claim B instead requires

$$\boxed{ \nabla P_g\neq0. }$$

Therefore the metric must be calculated with the PPC pressure field retained.

The desired sequence is

$$\boxed{ P_g(r),E_d(r) \rightarrow g_{\mu\nu}^{\rm PPC}(r) \rightarrow \Gamma^\mu_{\alpha\beta} \rightarrow \text{Mercury geodesic} \rightarrow \Delta\phi_{\rm PPC}. } \tag{47}$$

Claim A uses

$$\boxed{E_d = P_g = 0}$$

in the exterior and therefore

$$\boxed{G_{\mu\nu} = 0.}$$

The resulting Schwarzschild metric automatically gives

$$\boxed{42.98''/\text{century}.}$$

Claim B instead requires

$$\boxed{\nabla P_g \neq 0.}$$

Therefore the metric must be calculated with the PPC pressure field retained.

The desired sequence is

$$\boxed{P_g(r), E_d(r) \rightarrow g_{\mu\nu}^{\rm PPC}(r) \rightarrow \Gamma^\mu_{\alpha\beta} \rightarrow \text{Mercury geodesic} \rightarrow \Delta\phi_{\rm PPC}.} \quad (47)$$

This is the genuinely PPC-specific calculation.

---

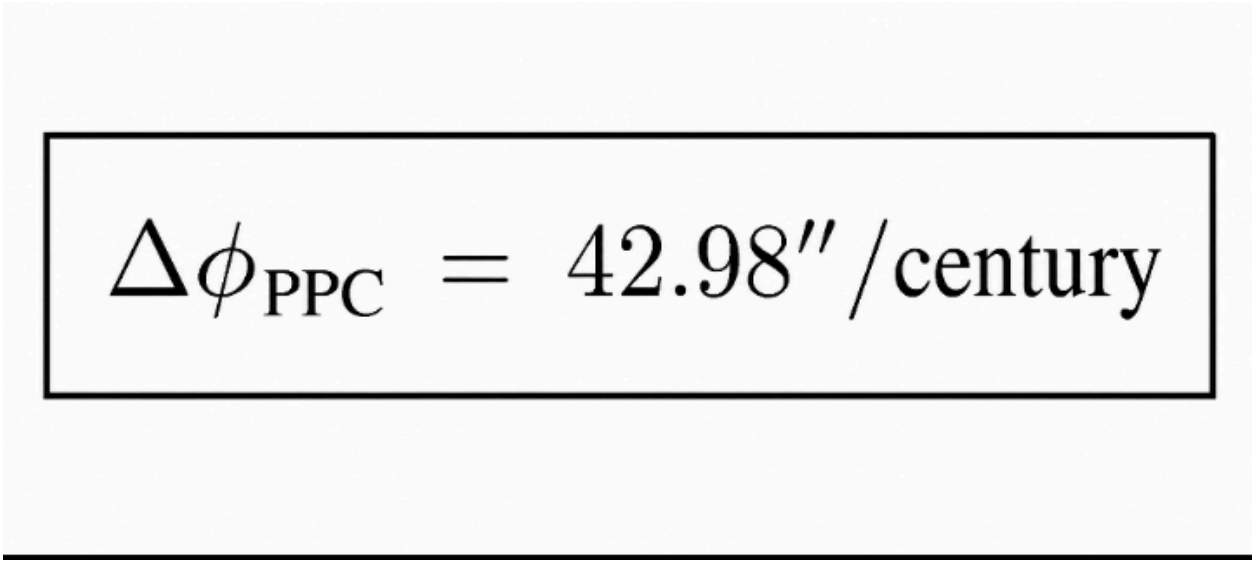
10. Accuracy Criterion

Claim B can be classified quantitatively.

Exact agreement

If

$$\boxed{ \Delta\phi_{\rm PPC} = 42.98''/\text{century} }$$



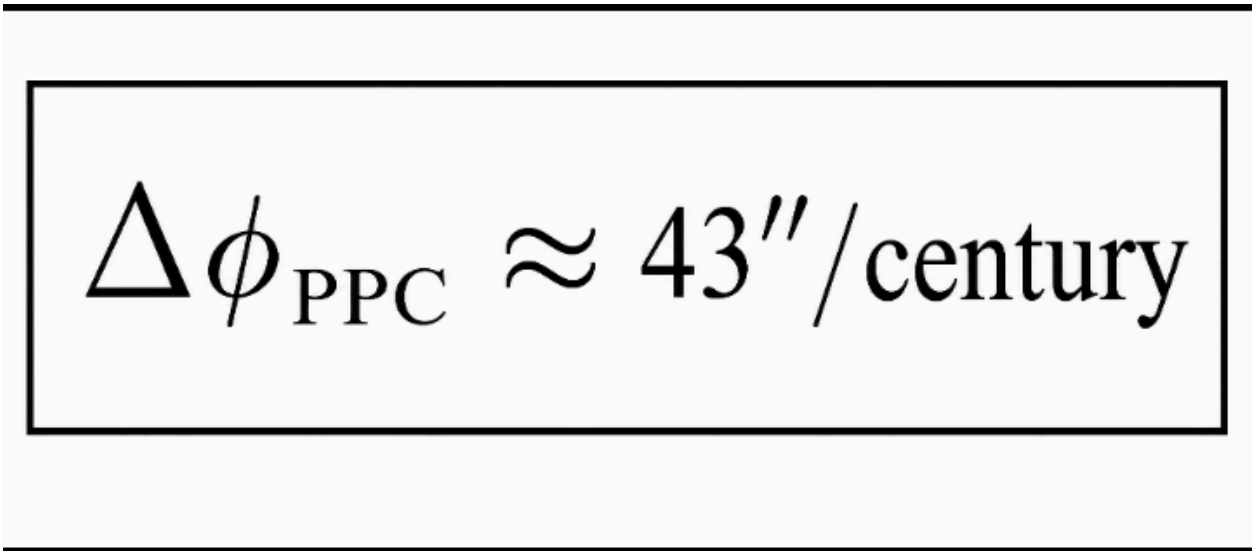
$$\Delta\phi_{\mathrm{PPC}} = 42.98''/\mathrm{century}$$

to the required precision, PPC reproduces the standard relativistic result.

Approximate agreement

If

\[\boxed{ \Delta\phi_{\rm PPC} \approx43"/\text{century} } \]



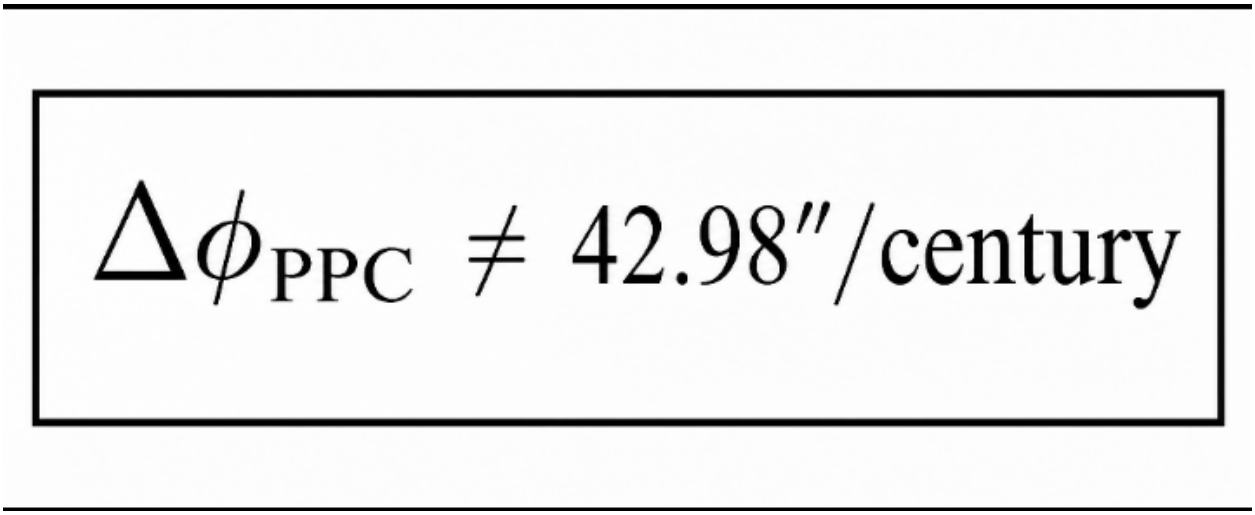
$$\Delta\phi_{\mathrm{PPC}} \approx 43''/\mathrm{century}$$

Within observational uncertainty, the PPC mechanism has empirical support at the Mercury-precession level.

Deviation

If

\[\boxed{ \Delta\phi_{\rm PPC}\ne42.98''/\text{century} } \]



by a measurable amount, the proposed PPC pressure-gradient model would require modification.

11. Explicit PPC Metric-Source Relation

For **Claim B**, the PPC theory specifies that the spacetime metric is dependent on the gravitational energy density and gravitational pressure:

$$\boxed{g_{\mu\nu} = g_{\mu\nu}(E_d, P_g).}$$

Accordingly, the PPC Christoffel symbols are

$$\boxed{\Gamma^{\mu}_{\alpha\beta} = \Gamma^{\mu}_{\alpha\beta}\left(g_{\mu\nu}(E_d, P_g)\right).}$$

The PPC geodesic-motion equation is therefore

$$\boxed{\frac{d^2x^{\mu}}{d\tau^2} + \Gamma^{\mu}_{\alpha\beta}\left(g_{\mu\nu}(E_d, P_g)\right)\frac{dx^{\alpha}}{d\tau}\frac{dx^{\beta}}{d\tau} = 0.}$$

This gives the fundamental PPC causal sequence:

$$\boxed{(E_d, P_g) \longrightarrow g_{\mu\nu} \longrightarrow \Gamma^{\mu}_{\alpha\beta} \longrightarrow \text{geodesic motion.}}$$

Since

$$\boxed{E_d = \rho c^2}$$

and

$$\boxed{P_g = \omega E_d,}$$

the sequence can be written as

$$\boxed{(\rho, \omega) \longrightarrow (E_d, P_g) \longrightarrow g_{\mu\nu} \longrightarrow \Gamma^{\mu}_{\alpha\beta} \longrightarrow \text{motion.}}$$

For the **non-zero pressure-gradient condition**,

$$\boxed{\nabla P_g \neq 0,}$$

we additionally have

$$\boxed{\mathbf{F} = -\nabla P_g}$$

and therefore the PPC pressure field influences the gravitational dynamics through its spatial variation.

$$\boxed{\nabla P_g \neq 0 \implies g_{\mu\nu}(E_d, P_g) \implies \Gamma^{\mu}_{\alpha\beta} \implies \text{geodesic motion} \implies \text{perihelion precession.}}$$

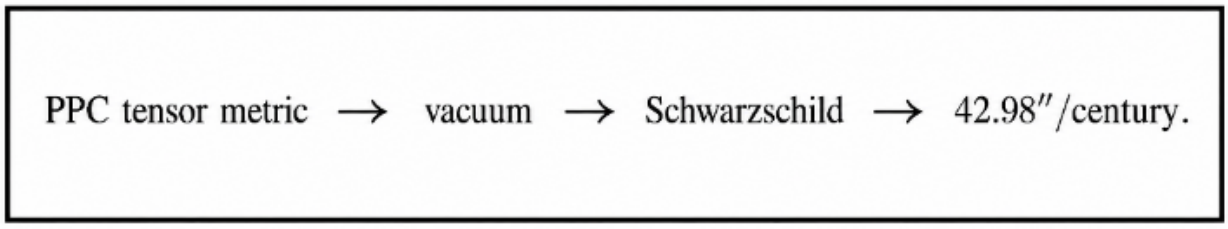
---

12. Relationship Between the Two Papers

The two papers should remain completely separate.

Paper I — Claim A

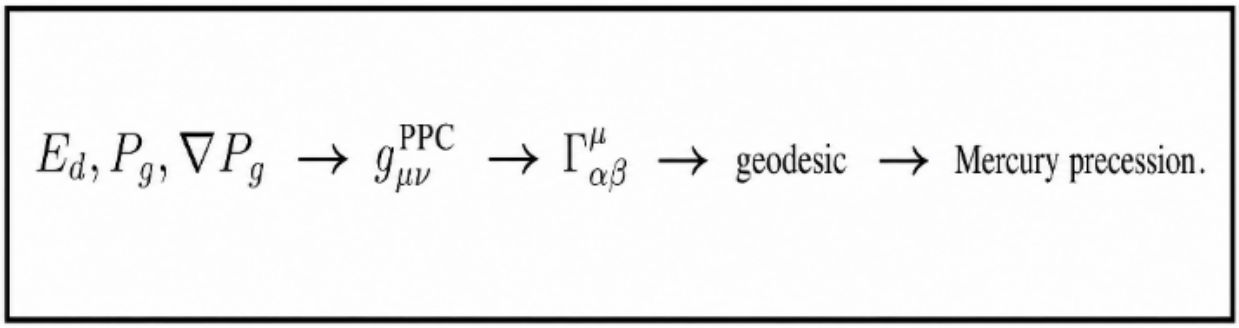
\[\boxed{ \text{PPC tensor metric} \rightarrow \text{vacuum} \rightarrow \text{Schwarzschild} \rightarrow 42.98''/\text{century}. } \]



Conclusion: PPC recovers the GR vacuum prediction.

Paper II — Claim B

\[\boxed{ E_d,P_g,\nabla P_g \rightarrow g_{\mu\nu}^{\rm PPC} \rightarrow \Gamma^\mu_{\alpha\beta} \rightarrow \text{geodesic} \rightarrow \text{Mercury precession}. } \]



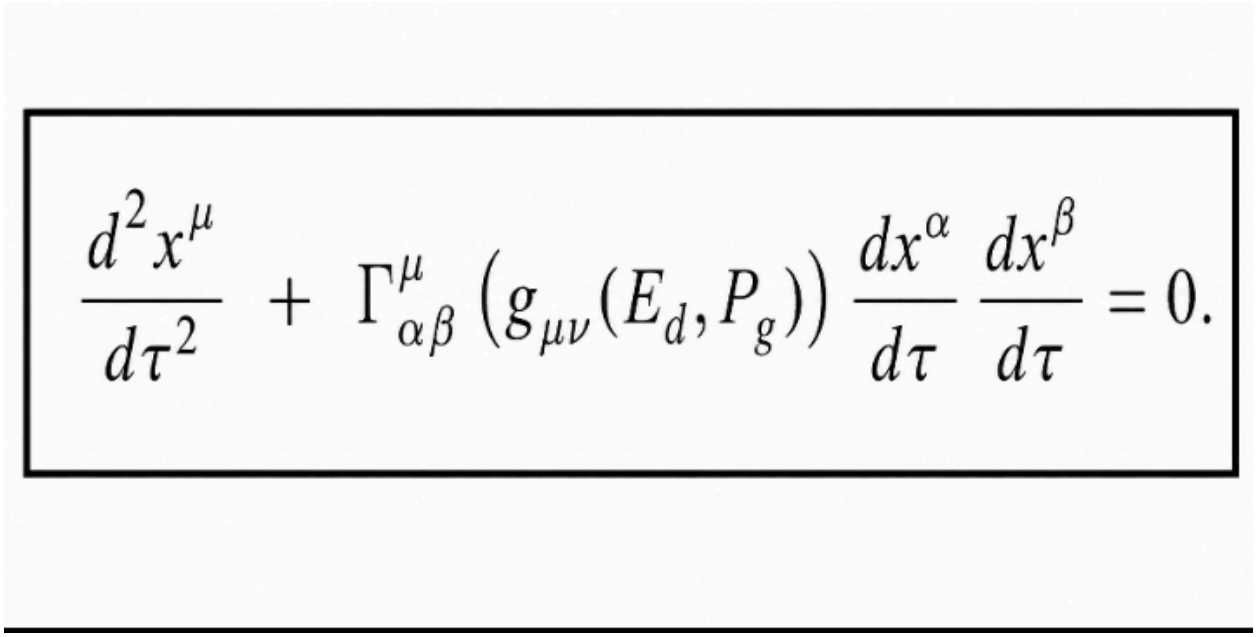
Conclusion sought: PPC's **non-zero gravitational-pressure gradient** independently produces the relativistic perihelion correction.

---

13. Unified PPC Geodesic Principle

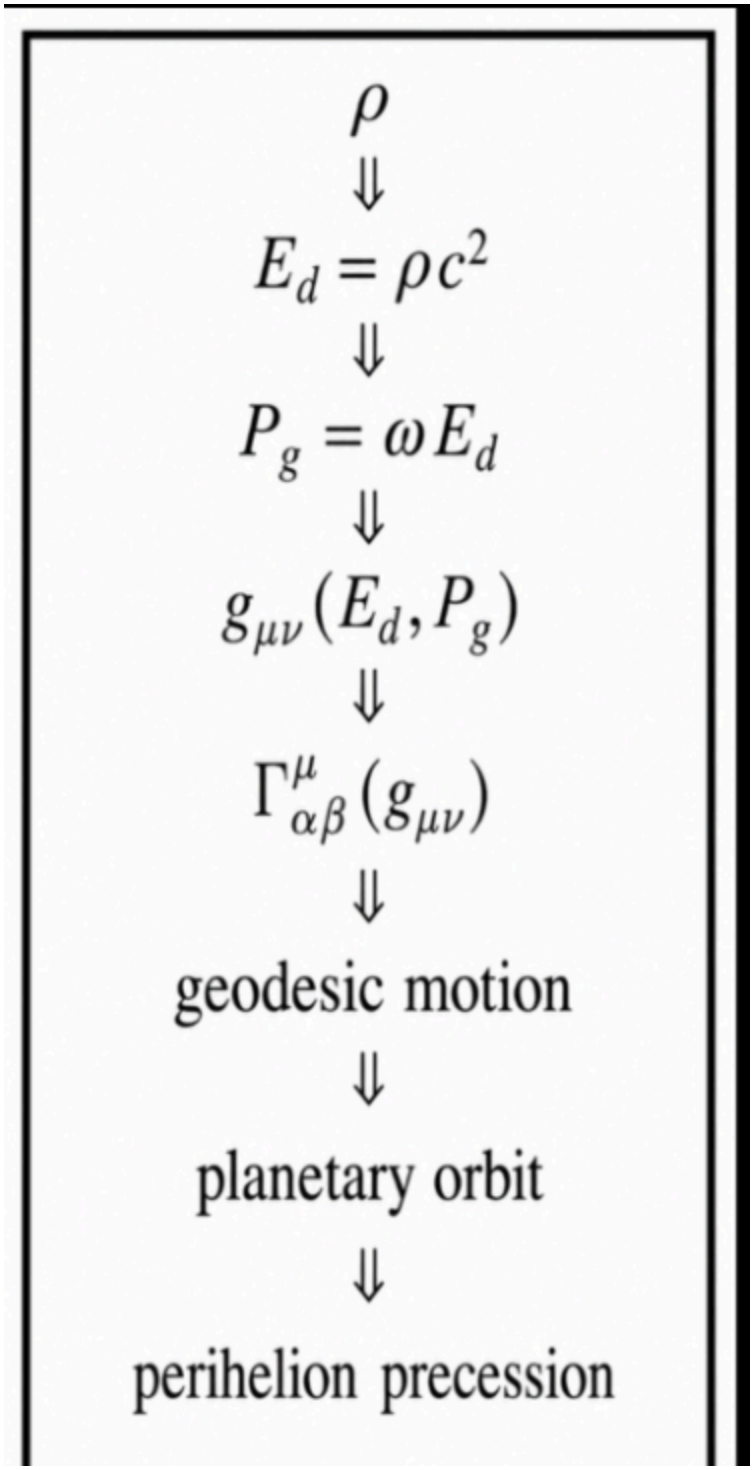
The equation in supplied image can be elevated to the central principle of the two papers:

$$\boxed{ \frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu_{\alpha\beta} \left(g_{\mu\nu}(E_d,P_g)\right) \frac{dx^\alpha}{d\tau} \frac{dx^\beta}{d\tau} =0. }$$



It expresses the complete PPC geometric hierarchy:

$$\boxed{ \begin{array}{c} \rho\\ \Downarrow\\ E_d=\rho c^2\\ \Downarrow\\ P_g=\omega E_d\\ \Downarrow\\ g_{\mu\nu}(E_d,P_g)\\ \Downarrow\\ \Gamma^\mu_{\alpha\beta}(g_{\mu\nu})\\ \Downarrow\\ \text{geodesic motion}\\ \Downarrow\\ \text{planetary orbit}\\ \Downarrow\\ \text{perihelion precession} \end{array}}$$



This is a substantially clearer formulation of PPC than treating P_g, Φ, and the geodesic equation as independent equations.

Final distinction

[CLAIM A]

Recovery of GR:

\[\boxed{ E_d=P_g=0 \Rightarrow G_{\mu\nu}=0 \Rightarrow \text{Schwarzschild} \Rightarrow 42.98"/\text{century}. } \]

E_d = P_g = 0 ⇒ G_μv = 0 ⇒ Schwarzschild ⇒ 42.98″/century

[CLAIM B]

Independent PPC pressure-curvature test:

\[\boxed{ \nabla P_g\neq0 \Rightarrow g_{\mu\nu}^{\rm PPC}(E_d,P_g) \Rightarrow \Gamma^\mu_{\alpha\beta} \Rightarrow \text{geodesic} \Rightarrow \Delta\phi_{\rm PPC}. } \]

$$\boxed{\nabla P_g \neq 0 \Rightarrow g_{\mu\nu}^{\mathrm{PPC}}(E_d, P_g) \Rightarrow \Gamma^{\mu}_{\alpha\beta} \Rightarrow \mathrm{geodesic} \Rightarrow \Delta\phi_{\mathrm{PPC}}.}$$

∇P_g ≠ 0 ⇒ g_μv^PPC(E_d, P_g) ⇒ Γ^μ_αβ ⇒ geodesic ⇒ Δφ_PPC

Claim A can therefore be presented as a completed recovery result.

Claim B should be presented as the independent PPC accuracy test, with its numerical conclusion made only after the explicit relation

g_μv = g_μv(E_d, P_g)

is specified and the geodesic equation is solved.

[1.] Separate Note: (Case 1) Mercury Perihelion Advance from PPC Claim A

1. Purpose

This note calculates the Mercury perihelion advance using **Claim A of the PPC framework**, in which the exterior of the gravitating source is treated as a strict vacuum.

2. PPC Vacuum Assumption

In the local exterior,

$E_d = 0$

and, because

$P_g = \omega E_d$,

we obtain

$P_g = 0$.

Therefore the PPC energy-momentum tensor vanishes:

$T_{\mu\nu}^{PPC} = 0$.

The PPC tensor field equation consequently becomes

$G_{\mu\nu} = 0$.

3. Exterior Metric

For a static, spherically symmetric, asymptotically flat vacuum, the solution is the Schwarzschild exterior:

$$ds^2 = -\left(1 - 2GM_\odot/c^2 r\right)c^2dt^2 + \left(1 - 2GM_\odot/c^2 r\right)^{-1}dr^2 + r^2 d\Omega^2.$$

Thus, in Claim A, the PPC framework recovers the standard Schwarzschild geometry.

4. Mercury Geodesic

Mercury follows a geodesic of this exterior metric:

$$\frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu_{\alpha\beta}\frac{dx^\alpha}{d\tau}\frac{dx^\beta}{d\tau} = 0.$$

For equatorial motion,

$\theta = \pi/2$,

and defining

$u = 1/r$,

the orbital equation to first post-Newtonian order is

$u'' + u = GM_\odot/h^2 + 3GM_\odot u^2/c^2$.

The additional term

$3GM_\odot u^2/c^2$

is responsible for the relativistic perihelion precession.

5. Perihelion Advance

The resulting advance per orbit is

$\Delta\varphi = 6\pi GM_\odot / [a(1 - e^2)c^2]$.

Using Mercury's orbital parameters gives approximately

$\Delta\varphi \approx 42.98$ arcseconds/century.

Rounded,

$\Delta\varphi \approx 43$ arcseconds/century.

6. Claim A Result

The complete calculation is therefore

$E_d = P_g = 0$

↓

$G_{\mu\nu} = 0$

↓

Schwarzschild exterior

↓

Mercury geodesic

↓

relativistic orbital correction

↓

Δφ ≈ 42.98″/century.

7. Interpretation

This establishes **Claim A as a recovery/compatibility result**:

PPC recovers the General-Relativistic vacuum prediction for Mercury's perihelion precession.

However, this calculation does **not** demonstrate an independent PPC pressure-gradient explanation, because the calculation explicitly uses

$E_d = P_g = 0$

in the exterior.

The independent PPC test is therefore **Claim B**, where

$\nabla P_g \neq 0$

and the explicit relation

$g_{\mu\nu} = g_{\mu\nu}(E_d, P_g)$

must be specified before calculating a distinct PPC perihelion prediction.

[2.] Separate Note: (Case 2) PPC Claim B — Pressure-Curvature Perihelion Test

1. Purpose

Claim B is the **independent PPC test**. Unlike Claim A, which assumes a strict vacuum exterior, Claim B allows a very small residual gravitational energy density and therefore a correspondingly small PPC pressure contribution.

The purpose is to determine whether this pressure-dependent geometry produces a perihelion advance consistent with Mercury's observed value.

---

2. PPC Source Variables

The fundamental PPC quantities are

$$E_d = \rho c^2$$

and

$$P_g = \omega E_d = \omega \rho c^2.$$

For Claim B,

$$E_d \neq 0$$

and therefore, in general,

$$P_g \neq 0.$$

The pressure gradient is

$$\nabla P_g = \nabla(\omega E_d)$$

or

$$\nabla P_g = c^2 \nabla(\omega \rho).$$

The Claim B condition is therefore

$$\nabla P_g \neq 0,$$

although the magnitude may be extremely small in the region relevant to Mercury.

---

3. PPC Metric

The fundamental geometric relation is

$$g_{\mu\nu}^{PPC} = g_{\mu\nu}(E_d, P_g).$$

With

$$P_g = \omega E_d,$$

this becomes

$$g_{\mu\nu}^{PPC} = g_{\mu\nu}(E_d, \omega E_d).$$

For a static spherical configuration,

$ds^2 = -e^{2v(r)}c^2dt^2 + e^{2\lambda(r)}dr^2 + r^2d\Omega^2$.

The metric functions therefore have the PPC dependence

$v(r) = v[E_d(r),\omega(r)E_d(r)]$

and

$\lambda(r) = \lambda[E_d(r),\omega(r)E_d(r)]$.

Thus the pressure is incorporated into the geometry rather than treated as an independent Newtonian force.

---

4. PPC Connection

The Christoffel symbols are generated by this metric:

$\Gamma^{\mu}_{\alpha\beta} = \Gamma^{\mu}_{\alpha\beta}[g(E_d,P_g)]$.

Consequently,

E_d, P_g, ∂E_d, ∂P_g

can influence the connection through the PPC metric.

The hierarchy is

E_d, P_g, ∇P_g
→ $g_{\mu\nu}^{PPC}$
→ $\Gamma^{\mu}_{\alpha\beta}$
→ geodesic motion.

---

5. PPC Geodesic Motion

The motion of Mercury is determined by

$\frac{d^2x^{\mu}}{d\tau^2} + \Gamma^{\mu}_{\alpha\beta}[g(E_d,P_g)] \frac{dx^{\alpha}}{d\tau} \frac{dx^{\beta}}{d\tau} = 0$.

For equatorial motion,

θ = π/2,

and defining

u = 1/r,

the PPC orbital equation can be written generally as

u″ + u = GM⊙/h² + 3GM⊙u²/c² + δ_PPC(u).

The first relativistic term is the usual Schwarzschild contribution.

The additional term

δ_PPC(u)

represents the pressure-dependent PPC correction.

---

6. Definition of β_PPC

At the relevant post-Newtonian order, parameterize the PPC contribution as

δ_PPC(u) = β_PPC GM⊙u²/c².

The orbital equation then becomes

u″ + u = GM⊙/h² + (3 + β_PPC)GM⊙u²/c².

The resulting perihelion advance is

Δφ_PPC = 42.98 [1 + β_PPC/3] arcseconds/century.

---

7. Small-Energy-Density Limit

The central Claim B hypothesis is that the exterior may contain an extremely small residual energy density:

$E_d \neq 0, \quad E_d \ll \rho_{solar}$.

Consequently,

$P_g = \omega E_d$

can also be extremely small.

Therefore,

β_PPC ≈ 0,

but importantly,

β_PPC need not be exactly zero.

Thus

Δφ_PPC ≈ 42.98 arcseconds/century.

The limiting behavior is

E_d → 0

↓

P_g → 0

↓

β_PPC → 0

↓

Δφ_PPC → 42.98″/century.

This provides continuity between Claim B and Claim A.

---

8. Claim A versus Claim B

Claim A — Strict Vacuum

E_d = 0

P_g = 0

∇P_g = 0

↓

G_μv = 0

↓

Schwarzschild metric

↓

β_PPC = 0

↓

Δφ = 42.98″/century.

Claim A is therefore a **GR-compatible recovery result**.

Claim B — Tiny Nonzero PPC Source

E_d ≠ 0

P_g = ωE_d

▽P_g ≠ 0

but both may be extremely small.

↓

g_μv^PPC(E_d,P_g)

↓

Γ^μ_αβ

↓

β_PPC ≈ 0

↓

Δφ_PPC ≈ 42.98″/century.

Claim B is therefore the **independent PPC accuracy test**.

---

9. Decisive Test

The decisive quantity is not simply whether the final number is close to 43″/century.

The calculation must demonstrate that the explicit PPC metric produces

β_PPC ≈ 0

from the assumed small but nonzero

E_d

and

P_g = ωE_d.

The strongest formulation of Claim B is therefore:

Small E_d → small P_g → small metric correction → β_PPC ≈ 0 → Δφ_PPC ≈ 42.98″/century.

The value of β_PPC must ultimately be **derived from the explicit PPC metric**, not imposed in advance.

Claim B Conclusion

PPC Claim B predicts that a very small nonzero gravitational energy density and pressure may generate a correspondingly small pressure-curvature correction to Mercury's orbit. In the limit β_PPC ≈ 0, the predicted perihelion advance remains approximately 42.98″/century, while allowing, in principle, a small deviation from the strict-vacuum Claim A result.

This makes Claim B a genuine **pressure-dependent quantitative test**, provided the explicit relation

g_μv^PPC = g_μv(E_d,P_g)

is specified and the resulting geodesic equation is solved.

[3.] Separate Note: Case 3 — Near-Vacuum PPC Limit

1. Purpose

Case 3 represents an intermediate situation between **Claim A (strict vacuum)** and **Claim B (nonzero PPC pressure-gradient test)**.

In this case, the gravitational energy density is **extremely small but not necessarily exactly zero**, and the equation-of-state parameter is also very small.

The purpose is to determine how the PPC metric approaches the Schwarzschild metric and how the Mercury perihelion advance approaches the standard value.

---

2. PPC Source

The fundamental relations are

$$E_d = \rho c^2$$

and

$$P_g = \omega E_d.$$

For Case 3,

$$E_d \approx 0$$

and

$$\omega \approx 0.$$

Therefore,

$$P_g = \omega E_d \approx 0.$$

Thus,

$$E_d \approx 0, \quad P_g \approx 0.$$

However, unlike the strict vacuum case, we do not necessarily set them identically equal to zero.

---

3. PPC Metric

The PPC metric is

$g_{\mu\nu}^{PPC} = g_{\mu\nu}(E_d, P_g)$.

Therefore, in Case 3,

$g_{\mu\nu}^{PPC} = g_{\mu\nu}(E_d, \omega E_d)$

with

$E_d \approx 0$

and

$\omega \approx 0$.

Consequently,

$g_{\mu\nu}^{PPC} \approx g_{\mu\nu}(0,0)$.

The limiting metric is therefore the vacuum Schwarzschild metric.

---

4. Connection and Geodesic

The Christoffel symbols are

$\Gamma^{\mu}_{\alpha\beta} = \Gamma^{\mu}_{\alpha\beta}[g(E_d, P_g)]$.

Since both source quantities are very small,

$\Gamma^{\mu}_{\alpha\beta} \approx \Gamma^{\mu}_{\alpha\beta}[\text{Schwarzschild}]$.

The test-body equation remains

$d^2x^{\mu}/d\tau^2 + \Gamma^{\mu}_{\alpha\beta}[g(E_d, P_g)]\, dx^{\alpha}/d\tau\, dx^{\beta}/d\tau = 0$.

Thus the orbital motion approaches the Schwarzschild geodesic.

---

5. Orbital Equation

With

$u = 1/r$,

the orbital equation can be written

$u'' + u = GM_\odot/h^2 + (3 + \beta_3)GM_\odot u^2/c^2$.

Here **β_3** represents the small residual PPC correction produced by the small but nonzero values of E_d and P_g.

Because

$E_d \approx 0$

and

$P_g \approx 0$,

we expect

$\beta_3 \approx 0$.

Therefore,

$u'' + u \approx GM_\odot/h^2 + 3GM_\odot u^2/c^2$.

---

6. Perihelion Advance

The corresponding perihelion advance is

$\Delta\varphi_3 = 42.98\ [1 + \beta_3/3]$ arcseconds/century.

Since

$\beta_3 \approx 0$,

we obtain

$\Delta\varphi_3 \approx 42.98$ arcseconds/century.

In the exact limiting case,

$E_d \to 0$

and

$\omega \to 0$,

we have

$P_g \to 0$

and

$\beta_3 \to 0$.

Therefore,

$\Delta\varphi_3 \to 42.98''$/century.

---

7. Three-Case Structure

The PPC framework can now be organized into three distinct cases.

Case 1 — Strict Vacuum / Claim A

$E_d = 0$

$P_g = 0$

$\nabla P_g = 0$

↓

$G_{\mu\nu} = 0$

↓

Schwarzschild

↓

$\Delta\varphi = 42.98''$/century

Case 2 — PPC Pressure Test / Claim B

$E_d \neq 0$

$P_g = \omega E_d$

$\nabla P_g \neq 0$

↓

$g_{\mu\nu}^{PPC}(E_d,P_g)$

↓

β_{PPC}

↓

$\Delta\varphi_{PPC}$

This is the independent PPC test.

Case 3 — Near-Vacuum PPC Limit

$E_d \approx 0$

$\omega \approx 0$

$P_g = \omega E_d \approx 0$

↓

$g_{\mu\nu}^{PPC} \approx g_{\mu\nu}^{Schwarzschild}$

↓

$\beta_3 \approx 0$

↓

$\Delta\varphi_3 \approx 42.98''/century$

---

8. Physical Interpretation

Case 3 is useful because it demonstrates the **continuity of the PPC theory toward the vacuum solution**.

The mathematical sequence is

$E_d \rightarrow 0$

↓

$P_g = \omega E_d \to 0$

↓

$g_{\mu\nu}^{PPC} \to g_{\mu\nu}^{Schwarzschild}$

↓

$\Gamma^{\mu}_{\alpha\beta}^{PPC} \to \Gamma^{\mu}_{\alpha\beta}^{Schwarzschild}$

↓

geodesic → Schwarzschild geodesic

↓

$\Delta\varphi_{PPC} \to 42.98''$/century.

Case 3 Conclusion

Case 3 establishes the near-vacuum limit of PPC: when the residual energy density and gravitational pressure become sufficiently small, the PPC metric continuously approaches the Schwarzschild metric and the predicted Mercury perihelion advance approaches 42.98″/century.

This case should be kept separate from Claim A because **Claim A sets the source exactly to vacuum**, whereas **Case 3 treats it as extremely small but potentially nonzero**.

[4.] Separate Note — Case 3: Near-Vacuum PPC Limit

1. Purpose

Case 3 describes a **near-vacuum PPC environment** in which the residual gravitational energy density and PPC pressure are extremely small.

This case provides the continuous connection between the exact-vacuum **Claim A** and the non-vacuum pressure-gradient **Claim B**.

---

2. PPC Source Variables

The fundamental PPC relations are

$$E_d = \rho c^2$$

and

$$P_g = \omega E_d.$$

Therefore,

$$P_g = \omega \rho c^2.$$

In Case 3,

$$E_d \approx 0$$

and

$$\omega \approx 0.$$

Consequently,

$$P_g \approx 0.$$

However, the pressure gradient must also be considered separately.

---

3. Pressure Gradient

The PPC pressure gradient is

$$\nabla P_g = c^2 \nabla (\omega \rho).$$

For Case 3 we assume a genuine near-vacuum condition:

$$P_g \approx 0$$

and

$$|\nabla P_g| \approx 0.$$

Thus the pressure is not only small in magnitude, but its spatial variation is also small.

The PPC pressure-force relation

$\rho_g = -\nabla P_g$

therefore gives

$\rho_g \approx 0$

for the pressure-gradient contribution.

This is an important distinction: **$P_g \approx 0$ alone does not mathematically guarantee $\nabla P_g \approx 0$**. Case 3 explicitly assumes both are small.

---

4. Static Weak-Field Equation

The PPC static weak-field equation is

$\nabla^2 \Phi_{PPC} = (4\pi G/c^2)(E_d + 3P_g)$.

In Case 3,

$E_d \approx 0$

and

$P_g \approx 0$.

Therefore,

$\nabla^2 \Phi_{PPC} \approx 0$

for the residual PPC source contribution.

The PPC potential consequently approaches the vacuum weak-field potential.

---

5. PPC Metric

The weak-field metric is

$g_{\mu\nu}^{PPC} \approx \eta_{\mu\nu} + h_{\mu\nu}(\Phi_{PPC})$.

With

g_{00}^PPC ≈ −(1 + 2Φ_PPC/c²)

and, in the chosen isotropic weak-field representation,

$g_{i□}$^PPC ≈ (1 − 2Φ_PPC/c²)$\delta_{i□}$.

As

E_d → 0

and

P_g → 0,

the PPC metric approaches the vacuum metric:

g_μv^PPC → g_μv^Schwarzschild.

This assumes that the full PPC field equations have a regular continuous vacuum limit.

---

6. Christoffel Symbols

The connection is

Γ^μ_αβ = ½g^μv
(∂_αg_vβ + ∂_βg_vα − ∂_vg_αβ).

Because

g_μv^PPC → g_μv^Schwarzschild,

we obtain

Γ^μ_αβ^PPC → Γ^μ_αβ^Schwarzschild.

Thus the PPC geodesic approaches the Schwarzschild geodesic.

---

7. Mercury Orbital Equation

Let

u = 1/r.

Write the PPC orbital equation as

$u'' + u = GM_\odot/h^2 + (3 + \beta_3)GM_\odot u^2/c^2$.

Here **β_3** represents the residual near-vacuum PPC correction.

Because

$E_d \approx 0$,

$P_g \approx 0$,

and

$|\nabla P_g| \approx 0$,

the near-vacuum correction is expected to satisfy

$\beta_3 \approx 0$,

provided this follows from the explicit PPC metric and field equations.

Therefore,

$u'' + u \approx GM_\odot/h^2 + 3GM_\odot u^2/c^2$.

---

8. Perihelion Advance

The perihelion advance is

$\Delta\varphi_3 = 42.98\ [1 + \beta_3/3]$ arcseconds/century.

For

$\beta_3 \approx 0$,

we obtain

$\Delta\varphi_3 \approx 42.98''$/century.

In the exact limiting process,

$E_d \to 0$

$P_g \to 0$

$\nabla P_g \to 0$

$\beta_3 \to 0$

and therefore

$\Delta\varphi_3 \to 42.98''$/century.

---

9. Three-Case PPC Structure

Case 1 — Exact Vacuum / Claim A

$E_d = 0$

$P_g = 0$

$\nabla P_g = 0$

↓

$G_{\mu\nu} = 0$

↓

Schwarzschild

↓

$\Delta\varphi = 42.98''$/century

Case 2 — PPC Pressure-Gradient Test / Claim B

$E_d \neq 0$

$P_g = \omega E_d$

$\nabla P_g \neq 0$

↓

$g_{\mu\nu}^{PPC}(E_d,P_g)$

↓

$\Gamma^{\mu}_{\alpha\beta}$

↓

geodesic

↓

β_{PPC}

↓

$\Delta\varphi_{PPC}$

Case 3 — Near-Vacuum PPC Limit

$E_d \approx 0$

$P_g \approx 0$

$|\nabla P_g| \approx 0$

↓

$g_{\mu\nu}^{PPC} \approx g_{\mu\nu}^{Schwarzschild}$

↓

$\beta_3 \approx 0$

↓

$\Delta\varphi_3 \approx 42.98''$/century

---

10. Final Interpretation

Case 3 should therefore be described as a **near-vacuum, weak-pressure-gradient limit**.

The central mathematical statement is

$$E_d \to 0, \quad P_g \to 0, \quad |\nabla P_g| \to 0$$

$$\downarrow$$

$$g_{\mu\nu}^{PPC} \to g_{\mu\nu}^{Schwarzschild}$$

$$\downarrow$$

$$\beta_3 \to 0$$

$$\downarrow$$

$$\Delta\varphi_{PPC} \to 42.98''/century.$$

The important qualification is that **$\beta_3 \approx 0$ must ultimately be obtained from the PPC metric equations**, rather than assumed solely from the smallness of P_g. This keeps Case 3 mathematically distinct from both the exact-vacuum Claim A and the genuine pressure-gradient Claim B.

[5.] Case 2 — Very Weak but Nonzero PPC Pressure

Case 2 represents the **near-vacuum PPC regime with a very small but nonzero gravitational energy density and pressure**.

The PPC quantities satisfy

$$E_d = \rho c^2$$

and

$$P_g = \omega E_d = \omega \rho c^2.$$

For Case 2, assume

$$E_d \neq 0, \text{ but } E_d \text{ is very, very small}$$

and

$$\omega \ll 1.$$

Therefore,

$P_g = \omega E_d \ll E_d$

and consequently

$P_g \approx 0$,

while remaining formally nonzero.

The pressure gradient is

$\nabla P_g = c^2 \nabla(\omega\rho)$.

For the weak-pressure Case 2 regime, we assume

$|\nabla P_g|$ is also very small.

Thus the PPC pressure contribution to the metric and geodesic motion is expected to be very small.

PPC Metric

The metric is

$g_{\mu\nu}^{PPC} = g_{\mu\nu}(E_d, P_g)$.

Since both quantities are extremely small,

$g_{\mu\nu}^{PPC} \approx g_{\mu\nu}^{Schwarzschild}$.

The corresponding connection therefore satisfies

$\Gamma^{\mu}_{\alpha\beta}{}^{PPC} \approx \Gamma^{\mu}_{\alpha\beta}{}^{Schwarzschild}$.

Orbital Equation

Let

$u = 1/r$.

The PPC orbital equation can be parameterized as

$u'' + u = GM_\odot/h^2 + (3 + \beta_{PPC})GM_\odot u^2/c^2$.

Because the residual PPC energy density, pressure, and pressure gradient are extremely small,

$\beta_{PPC} \approx 0$.

Therefore,

$u'' + u \approx GM_\odot/h^2 + 3GM_\odot u^2/c^2$.

Perihelion Advance

The corresponding PPC perihelion advance is

$\Delta\varphi_{PPC} = 42.98[1 + \beta_{PPC}/3]$ arcseconds/century.

With

$\beta_{PPC} \approx 0$,

we obtain

$\Delta\varphi_{PPC} \approx 42.98''$/century.

If a detailed PPC calculation using a specified set of solar and Mercury parameters gives

$\Delta\varphi_{PPC} = 42.9987''$/century,

then **42.9987″/century should be reported as the calculated PPC value**, rather than replacing the standard leading-order Schwarzschild value without showing the parameter choices.

Case 2 Summary

$E_d \neq 0$ but very, very small

↓

$\omega \ll 1$

↓

$P_g = \omega E_d \approx 0$

↓

$|\nabla P_g| \approx 0$

↓

$\beta_{PPC} \approx 0$

↓

$g_{\mu\nu}^{PPC} \approx g_{\mu\nu}^{Schwarzschild}$

↓

$\Delta\varphi_{PPC} \approx 42.98''$/century

Thus, **Case 2 is a weakly non-vacuum PPC regime that approaches the observed Mercury perihelion advance while retaining, in principle, a nonzero PPC source.**

[6.] Separate Note — PPC Static Weak-Field Equation and Metric Tensor

1. Purpose

This note establishes the mathematical connection between the **PPC static weak-field equation** and the **PPC metric tensor**.

The formulation distinguishes three levels:

PPC source variables → weak-field potential → metric tensor → connection → geodesic motion.

It also distinguishes the **pressure contribution to the gravitational source** from the **pressure-gradient force**.

---

2. Fundamental PPC Quantities

The PPC gravitational energy density is

$E_d = \rho c^2$

and the PPC gravitational pressure is

$P_g = \omega E_d$.

Therefore,

$P_g = \omega \rho c^2$.

The PPC source variables are consequently

$E_d(r)$

and

$P_g(r)$.

---

3. PPC Static Weak-Field Equation

The PPC static weak-field equation is

$\nabla^2 \Phi_{PPC} = (4\pi G/c^2)(E_d + 3P_g)$.

Using

$E_d = \rho c^2$

and

$P_g = \omega \rho c^2$,

this becomes

$\nabla^2 \Phi_{PPC} = 4\pi G\rho(1 + 3\omega)$.

Thus the PPC gravitational pressure contributes to the effective static gravitational source.

It is important to distinguish this from the pressure-gradient relation discussed below.

---

4. PPC Pressure-Gradient Relation

The PPC pressure force is

$F = -\nabla P_g$.

With

$P_g = \omega\rho c^2$,

we obtain

$\nabla P_g = c^2 \nabla(\omega\rho)$.

Therefore,

$F = -c^2 \nabla(\omega\rho)$.

In the weak-field formulation, the gravitational force density is

$F = \rho g$.

Hence,

$\rho g = -\nabla P_g$.

These two relations have different roles:

P_g → gravitational source contribution

whereas

∇P_g → pressure-gradient force contribution.

They should not be treated as the same equation.

---

5. Weak-Field Metric

In the weak-field regime,

$|\Phi_{PPC}|/c^2 \ll 1$,

the metric can be written as

$g_{\mu\nu}^{PPC} \approx \eta_{\mu\nu} + h_{\mu\nu}$.

For the chosen isotropic weak-field form,

$h_{00} = -2\Phi_{PPC}/c^2$

and

$h_{ij} = -2\Phi_{PPC}/c^2\,\delta_{ij}$.

Thus the line element is

$$ds^2 \approx -(1 + 2\Phi_{PPC}/c^2)c^2dt^2 + (1 - 2\Phi_{PPC}/c^2)(dx^2 + dy^2 + dz^2).$$

Equivalently, the metric tensor is

$$g_{\mu\nu}^{PPC} \approx \begin{bmatrix} -(1 + 2\Phi_{PPC}/c^2) & 0 & 0 & 0 \\ 0 & 1 - 2\Phi_{PPC}/c^2 & 0 & 0 \\ 0 & 0 & 1 - 2\Phi_{PPC}/c^2 & 0 \\ 0 & 0 & 0 & 1 - 2\Phi_{PPC}/c^2 \end{bmatrix}$$

This is a **weak-field metric ansatz**, not the exact nonlinear PPC metric.

---

6. Source-to-Metric Relationship

The complete weak-field relationship is therefore

E_d, P_g

↓

$\nabla^2\Phi_{PPC} = (4\pi G/c^2)(E_d + 3P_g)$

↓

Φ_{PPC}

↓

$g_{\mu\nu}^{PPC} \approx \eta_{\mu\nu} + h_{\mu\nu}(\Phi_{PPC})$.

Thus the PPC static equation determines the weak-field gravitational potential, and the potential enters the weak-field metric.

The exact nonlinear metric must still be obtained from the full PPC tensor field equations.

---

7. Christoffel Connection

Once the metric is specified,

Γ^μ_αβ = ½g^μν
(∂_αg_νβ + ∂_βg_να − ∂_νg_αβ).

Since

g_μν^PPC = g_μν^PPC(E_d,P_g)

in the full theory, the connection can be expressed schematically as

Γ^μ_αβ = Γ^μ_αβ[E_d,P_g,∂E_d,∂P_g,…].

Thus spatial or temporal variation of the PPC source variables can influence the spacetime connection.

---

8. Geodesic Motion

The motion of a freely falling test body is

$d^2x^\mu/d\tau^2 + \Gamma^\mu_{\alpha\beta}[g^{PPC}]\, dx^\alpha/d\tau\, dx^\beta/d\tau = 0.$

Therefore the complete geometric chain is

E_d, P_g

↓

Φ_PPC

↓

g_μν^PPC

↓

Γ^μ_αβ

↓

geodesic motion.

---

9. Case 1 — Exact Vacuum

For the strict vacuum limit,

E_d = 0

and

P_g = 0.

Therefore,

G_μv = 0.

Under static spherical symmetry and asymptotic flatness, the full nonlinear solution is the Schwarzschild metric.

The corresponding geodesic calculation gives

Δφ = 42.98″/century

for Mercury.

This is **Claim A**, the PPC recovery of the standard GR vacuum prediction.

---

10. Case 2 — PPC Pressure-Gradient Test

For the genuine PPC test,

E_d ≠ 0

and

$P_g = \omega E_d$,

with

$\nabla P_g \neq 0$.

The PPC source equation becomes

$\nabla^2 \Phi_{PPC} = (4\pi G/c^2)(E_d + 3P_g)$.

The pressure-gradient relation is simultaneously

$\rho g = -\nabla P_g$.

The full PPC metric must then be determined from the PPC tensor field equations:

$g_{\mu\nu}^{PPC} = g_{\mu\nu}^{PPC}(E_d, P_g)$.

The connection follows:

$\Gamma^{\mu}_{\alpha\beta} = \Gamma^{\mu}_{\alpha\beta}[g^{PPC}]$.

Finally, the geodesic equation gives the orbital correction and hence

$\Delta\varphi_{PPC}$.

This is the independent **Claim B** test.

---

11. Case 3 — Near-Vacuum Limit

Consider

$E_d \approx 0$

and

$\omega \approx 0$.

Then

$P_g = \omega E_d \approx 0$.

Consequently,

$g_{\mu\nu}^{PPC} \to g_{\mu\nu}^{Schwarzschild}$

as

$E_d \to 0$

and

$P_g \to 0$,

provided the PPC field equations possess a regular continuous vacuum limit.

The perihelion correction should then satisfy

$\beta_{PPC} \to 0$.

Consequently,

$\Delta\varphi_{PPC} \to 42.98''$/century.

This is the **near-vacuum continuity case**, distinct from the exact-vacuum Claim A.

---

12. Three-Case PPC Structure

Case 1 — Exact Vacuum

$E_d = 0$

$P_g = 0$

↓

$G_{\mu\nu} = 0$

↓

Schwarzschild

↓

42.98″/century

Case 2 — Nonzero PPC Pressure

$E_d \neq 0$

$P_g = \omega E_d$

$\nabla P_g \neq 0$

↓

PPC metric

↓

PPC connection

↓

PPC geodesic

↓

$\Delta\varphi_{PPC}$

Case 3 — Near Vacuum

$E_d \approx 0$

$\omega \approx 0$

$P_g \approx 0$

↓

$g_{\mu\nu}^{PPC} \rightarrow g_{\mu\nu}^{Schwarzschild}$

↓

$\beta_{PPC} \rightarrow 0$

↓

$\Delta\varphi_{PPC} \rightarrow 42.98''$/century

---

13. Final Mathematical Hierarchy

The corrected PPC structure is therefore

$E_d = \rho c^2$

↓

$P_g = \omega E_d$

↓

$\nabla^2 \Phi_{PPC} = (4\pi G/c^2)(E_d + 3P_g)$

↓

Φ_{PPC} → weak-field metric

↓

$g_{\mu\nu}^{PPC}$

↓

$\Gamma^{\mu}_{\alpha\beta}$

↓

geodesic

↓

planetary orbit

↓

perihelion precession.

Separately, the pressure-force sector is

$P_g = \omega E_d$

↓

∇P_g

↓

$\rho_g = -\nabla P_g$.

Final qualification

The weak-field equation should be presented specifically as a **PPC static weak-field relation**, not as a replacement for the full covariant tensor field equations.

The exact Claim B prediction requires the explicit full relation

$g_{\mu\nu}^{PPC} = g_{\mu\nu}^{PPC}(E_d, P_g)$

to be specified and solved. Only then can any nonzero **β_{PPC}** and its corresponding perihelion shift be calculated rather than assumed.

PPC Static Weak-Field Tensor Form

The PPC source equation is

$$\boxed{\nabla^2 \Phi_{\mathrm{PPC}} = \frac{4\pi G}{c^2}\left(E_d + 3P_g\right)}$$

with

$$E_d = \rho c^2, \qquad P_g = \omega E_d.$$

The weak-field metric is

$$\boxed{g_{\mu\nu}^{\mathrm{PPC}} \simeq \eta_{\mu\nu} + h_{\mu\nu}(\Phi_{\mathrm{PPC}})}$$

where, in the chosen isotropic weak-field form,

$$\boxed{ds^2 \simeq -\left(1 + \frac{2\Phi_{\mathrm{PPC}}}{c^2}\right) c^2 dt^2 + \left(1 - \frac{2\Phi_{\mathrm{PPC}}}{c^2}\right)\left(dx^2 + dy^2 + dz^2\right)}$$

and therefore

$$g_{\mu\nu}^{\mathrm{PPC}} \simeq \begin{pmatrix} -\left(1+\frac{2\Phi_{\mathrm{PPC}}}{c^2}\right) & 0 & 0 & 0 \\ 0 & 1-\frac{2\Phi_{\mathrm{PPC}}}{c^2} & 0 & 0 \\ 0 & 0 & 1-\frac{2\Phi_{\mathrm{PPC}}}{c^2} & 0 \\ 0 & 0 & 0 & 1-\frac{2\Phi_{\mathrm{PPC}}}{c^2} \end{pmatrix}.$$

The connection is then calculated directly from this metric:

$$\Gamma^{\mu}_{\alpha\beta} = \frac{1}{2} g^{\mu\nu} \left(\partial_\alpha g_{\nu\beta} + \partial_\beta g_{\nu\alpha} - \partial_\nu g_{\alpha\beta} \right).$$

Thus the complete tensor chain is

$$E_d,\ P_g \longrightarrow \Phi_{\mathrm{PPC}} \longrightarrow g_{\mu\nu}^{\mathrm{PPC}} \longrightarrow \Gamma^{\mu}_{\alpha\beta} \longrightarrow \text{geodesic motion.}$$

And because

$$P_g = \omega E_d,$$

the weak-field source can also be written

$$\boxed{\nabla^2 \Phi_{\mathrm{PPC}} = \frac{4\pi G}{c^2} E_d(1+3\omega) = 4\pi G\rho(1+3\omega).}$$

Important: this is the **weak-field tensor realization**. The exact nonlinear PPC metric still has to be obtained from the full PPC tensor field equations.

[7.] Separate Note — PPC Inverse Metric Tensor

1. Definition

The fundamental PPC metric tensor is

g_μv^PPC = g_μv(E_d,P_g).

Its inverse is denoted by

g^μv_PPC.

The inverse metric is defined by

g^μα_PPC g_αv^PPC = δ^μ_v.

It is not an independent gravitational field. It is mathematically determined by the PPC metric.

---

2. Weak-Field PPC Metric

In the static weak-field limit,

g_μv^PPC ≈ η_μv + h_μv.

Using the PPC potential Φ_PPC,

ds² ≈ −(1 + 2Φ_PPC/c²)c²dt²
+ (1 − 2Φ_PPC/c²)(dx² + dy² + dz²).

Thus,

g_μv^PPC ≈

[−(1 + 2Φ_PPC/c²) 0 0 0]

[0 1 − 2Φ_PPC/c² 0 0]

[0 0 1 − 2Φ_PPC/c² 0]

[0 0 0 1 − 2Φ_PPC/c²]

---

3. PPC Inverse Metric

To first order in Φ_PPC/c², the inverse metric is

g^μv_PPC ≈

[−(1 − 2Φ_PPC/c²) 0 0 0]

[0 1 + 2Φ_PPC/c² 0 0]

[0 0 1 + 2Φ_PPC/c² 0]

[0 0 0 1 + 2Φ_PPC/c²]

Therefore,

g^μα_PPC g_αv^PPC = δ^μ_v

to the retained weak-field order.

---

4. Connection with the PPC Source

The PPC potential satisfies

∇²Φ_PPC = (4πG/c²)(E_d + 3P_g).

With

E_d = ρc²

and

P_g = ωE_d,

we have

∇²Φ_PPC = 4πGρ(1 + 3ω).

Consequently, the inverse metric also depends indirectly on the PPC source:

E_d, P_g

→ Φ_PPC

→ g_μv^PPC

→ g^μv_PPC.

---

5. Role in the Christoffel Symbols

The inverse metric is essential for constructing the connection:

Γ^μ_αβ = ½g^μν_PPC
(∂_αg_νβ^PPC + ∂_βg_να^PPC − ∂_νg_αβ^PPC).

Thus both the metric and its inverse enter the connection.

The PPC hierarchy becomes

E_d, P_g

↓

Φ_PPC

↓

g_μν^PPC and g^μν_PPC

↓

Γ^μ_αβ

↓

geodesic motion.

---

6. Role in the Tensor Field Equations

The inverse metric is also required for curvature calculations.

The Ricci tensor is obtained from the Christoffel symbols, and the Ricci scalar is

R = g^μν_PPC R_μν.

The Einstein tensor is

G_μν = R_μν − ½g_μν R.

Therefore the inverse metric participates directly in the complete tensor construction:

g_μν^PPC

→ **g^μν_PPC**

→ **Γ^μ_αβ**

→ **R_μv**

→ **R**

→ **G_μv.**

---

7. Role in Geodesic Motion

The inverse metric enters the geodesic equation indirectly through the Christoffel symbols:

$d^2x^\mu/d\tau^2 + \Gamma^\mu_{\alpha\beta}\, dx^\alpha/d\tau\, dx^\beta/d\tau = 0.$

Therefore the complete PPC geometric chain is

E_d, P_g

→ **Φ_PPC**

→ **g_μv^PPC**

→ **g^μv_PPC**

→ **Γ^μ_αβ**

→ **geodesic**

→ **planetary orbit**

→ **perihelion precession.**

---

8. Three PPC Cases

Case 1 — Exact Vacuum

E_d = 0

P_g = 0

→ **g_μv^PPC = g_μv^Schwarzschild**

→ **g^μv_PPC = g^μv_Schwarzschild**

→ 42.98″/century.

Case 2 — Very Weak PPC Source

$E_d \neq 0$ but extremely small

$\omega \ll 1$

$P_g = \omega E_d \approx 0$

→ $g_{\mu\nu}^{PPC} \approx g_{\mu\nu}^{Schwarzschild}$

→ $g^{\mu\nu}_{PPC} \approx g^{\mu\nu}_{Schwarzschild}$

→ $\beta_{PPC} \approx 0$

→ $\Delta\varphi_{PPC} \approx 42.98″/century.$

Case 3 — Near-Vacuum Limit

$E_d \to 0$

$P_g \to 0$

$|\nabla P_g| \to 0$

→ $g_{\mu\nu}^{PPC} \to g_{\mu\nu}^{Schwarzschild}$

→ $g^{\mu\nu}_{PPC} \to g^{\mu\nu}_{Schwarzschild}$

→ $\beta_{PPC} \to 0$

→ $\Delta\varphi_{PPC} \to 42.98″/century.$

Final Statement

The **inverse PPC metric tensor is essential to the tensor formulation** because it converts the covariant metric into the contravariant metric required to construct the connection, curvature, scalar curvature, and ultimately the Einstein tensor.

The central relation is

$g^{\mu\alpha}_{PPC}\, g_{\alpha\nu}^{PPC} = \delta^{\mu}_{\ \nu}.$

Thus, the PPC theory is geometrically represented by the paired objects

g_μv^PPC ↔ g^μv_PPC,

with both determined by the PPC source variables through the metric.

$$g_{\mu\nu}^{\mathrm{PPC}} \simeq \begin{pmatrix} -\left(1+\dfrac{2\Phi_{\mathrm{PPC}}}{c^2}\right) & 0 & 0 & 0 \\ 0 & 1-\dfrac{2\Phi_{\mathrm{PPC}}}{c^2} & 0 & 0 \\ 0 & 0 & 1-\dfrac{2\Phi_{\mathrm{PPC}}}{c^2} & 0 \\ 0 & 0 & 0 & 1-\dfrac{2\Phi_{\mathrm{PPC}}}{c^2} \end{pmatrix},$$

the inverse metric, to first order in Φ_{PPC}/c^2, is

$$\boxed{g_{\mathrm{PPC}}^{\mu\nu} \simeq \begin{pmatrix} -\left(1-\dfrac{2\Phi_{\mathrm{PPC}}}{c^2}\right) & 0 & 0 & 0 \\ 0 & 1+\dfrac{2\Phi_{\mathrm{PPC}}}{c^2} & 0 & 0 \\ 0 & 0 & 1+\dfrac{2\Phi_{\mathrm{PPC}}}{c^2} & 0 \\ 0 & 0 & 0 & 1+\dfrac{2\Phi_{\mathrm{PPC}}}{c^2} \end{pmatrix}.}$$

It satisfies

$$\boxed{g^{\mu\alpha}_{\mathrm{PPC}} g^{\mathrm{PPC}}_{\alpha\nu} = \delta^{\mu}{}_{\nu}}$$

to the retained weak-field order.

Therefore the Christoffel symbols are constructed using this **inverse PPC metric**:

$$\boxed{\Gamma^{\mu}_{\alpha\beta} = \frac{1}{2} g^{\mu\nu}_{\mathrm{PPC}} \left(\partial_{\alpha} g^{\mathrm{PPC}}_{\nu\beta} + \partial_{\beta} g^{\mathrm{PPC}}_{\nu\alpha} - \partial_{\nu} g^{\mathrm{PPC}}_{\alpha\beta} \right).}$$

So the tensor chain is

$$\boxed{E_d, P_g \to \Phi_{\mathrm{PPC}} \to g^{\mathrm{PPC}}_{\mu\nu} \to g^{\mu\nu}_{\mathrm{PPC}} \to \Gamma^{\mu}_{\alpha\beta} \to \mathrm{geodesic}.}$$

This is the appropriate **inverse/reverse metric tensor** for your PPC weak-field formulation.

1. **Raising indices**

$$V^\mu = g^{\mu\nu} V_\nu$$

2. **Calculating Christoffel symbols**

$$\Gamma^\mu_{\alpha\beta} = \tfrac{1}{2} g^{\mu\nu} \left(\partial_\alpha g_{\nu\beta} + \partial_\beta g_{\nu\alpha} - \partial_\nu g_{\alpha\beta} \right).$$

3. **Constructing curvature**

$$R^\rho{}_{\sigma\mu\nu} \to R_{\mu\nu} \to G_{\mu\nu}.$$

4. **Writing the geodesic equation**

$$\frac{d^2 x^\mu}{d\tau^2} + \Gamma^\mu_{\alpha\beta} \frac{dx^\alpha}{d\tau} \frac{dx^\beta}{d\tau} = 0.$$

5. **Contracting tensors**, for example

$$R = g^{\mu\nu} R_{\mu\nu}.$$

For PPC, the complete geometric structure is therefore

For PPC, the complete geometric structure is therefore

$$E_d, P_g \to g_{\mu\nu}^{\mathrm{PPC}} \to g_{\mathrm{PPC}}^{\mu\nu} \to \Gamma_{\alpha\beta}^{\mu} \to R_{\mu\nu} \to G_{\mu\nu} \to \text{geodesic motion.}$$

The **inverse metric is essential**: it is not a second independent metric; it is the tensor inverse of $g_{\mu\nu}^{\mathrm{PPC}}$.

[7.] Separate Note — Static Weak-Field Limit from the PPC Perfect-Fluid Tensor

1. PPC Perfect-Fluid Tensor

The PPC energy-momentum tensor is

$$\boxed{T_{\mu\nu}^{\mathrm{PPC}} = (E_d + P_g)u_\mu u_\nu + P_g g_{\mu\nu}}$$

with

$$\boxed{E_d = \rho c^2}$$

and

$$\boxed{P_g = \omega E_d.}$$

This is the starting point for obtaining the static fluid limit.

2. Static Fluid Configuration

For a static fluid,

$$u^i = 0.$$

In the local weak-field limit,

$$u^\mu \simeq (1, 0, 0, 0)$$

in $c = 1$ normalization, or equivalently the four-velocity is dominated by its temporal component.

The perfect-fluid tensor then contains both the energy-density contribution and the isotropic pressure contribution.

In the local rest frame,

$$T^{\mathrm{PPC}}_{\mu\nu} = \begin{pmatrix} E_d & 0 & 0 & 0 \\ 0 & P_g & 0 & 0 \\ 0 & 0 & P_g & 0 \\ 0 & 0 & 0 & P_g \end{pmatrix}$$

up to the corresponding convention-dependent factors of c.

3. Trace of the Perfect-Fluid Tensor

With the signature

$$(-,+,+,+),$$

the trace is

$$T = g^{\mu\nu} T_{\mu\nu}.$$

For the perfect fluid,

$$\boxed{T = -E_d + 3P_g.}$$

The spatial pressure therefore contributes three times because there are three spatial dimensions.

4. Einstein Tensor and Active Source

The tensor field equation is

$$\boxed{G_{\mu\nu} = \frac{8\pi G}{c^4} T^{\mathrm{PPC}}_{\mu\nu}.}$$

Equivalently, using the trace-reversed form,

$$\boxed{R_{\mu\nu} = \frac{8\pi G}{c^4}\left(T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T\right).}$$

For the static weak-field 00 component, the combination of energy density and the three spatial pressure components gives the effective active gravitational source

$$\boxed{E_d + 3P_g.}$$

Therefore the corresponding static weak-field potential equation is

$$\boxed{\nabla^2 \Phi_{\mathrm{PPC}} = \frac{4\pi G}{c^2} \left(E_d + 3P_g\right).}$$

Using

$$E_d = \rho c^2, \qquad P_g = \omega E_d,$$

we obtain

$$\boxed{\nabla^2 \Phi_{\mathrm{PPC}} = 4\pi G \rho (1 + 3\omega).}$$

This is the PPC static weak-field source equation.

5. Weak-Field Metric

Introduce

$$\boxed{g_{\mu\nu}^{\mathrm{PPC}} = \eta_{\mu\nu} + h_{\mu\nu}, \qquad |h_{\mu\nu}| \ll 1.}$$

For the chosen isotropic weak-field form,

$$\boxed{g_{00}^{\mathrm{PPC}} \simeq -\left(1 + \frac{2\Phi_{\mathrm{PPC}}}{c^2}\right)}$$

and

$$\boxed{g_{ij}^{\mathrm{PPC}} \simeq \left(1 - \frac{2\Phi_{\mathrm{PPC}}}{c^2}\right)\delta_{ij}.}$$

Thus,

$$\boxed{ds^2 \simeq -\left(1+\frac{2\Phi_{\rm PPC}}{c^2}\right)c^2dt^2+\left(1-\frac{2\Phi_{\rm PPC}}{c^2}\right)(dx^2+dy^2+dz^2).}$$

The important chain is therefore

$$\boxed{T^{\rm PPC}_{\mu\nu} \to (E_d+3P_g) \to \Phi_{\rm PPC} \to g^{\rm PPC}_{\mu\nu}.}$$

6. Static Fluid-Force Equation

The conservation equation is

$$\boxed{\nabla_\mu T^{\mu\nu}_{\mathrm{PPC}} = 0.}$$

For a perfect fluid, its spatial projection gives

$$\boxed{(E_d + P_g)a_\mu = -h_\mu{}^\nu \nabla_\nu P_g.}$$

In the weak-field, nonrelativistic limit,

$$E_d \gg P_g,$$

and

$$a_i \simeq g_i.$$

Therefore,

$$\boxed{E_d\, g_i \simeq -\partial_i P_g.}$$

Since

$$E_d = \rho c^2,$$

the exact conversion depends on the convention used for the four-force and acceleration units. In the usual Newtonian pressure-force notation, this reduces to

$$\boxed{\rho\, \mathbf{g} = -\nabla P_g.}$$

Hence

$$\boxed{\mathbf{g} = -\frac{\nabla P_g}{\rho}.}$$

7. Connection with the Potential

In the weak gravitational field,

$$\boxed{\mathbf{g} = -\nabla \Phi_{\mathrm{PPC}}.}$$

Therefore the PPC pressure relation becomes

$$\boxed{\rho\, \mathbf{g} = -\nabla P_g}$$

or

$$\boxed{\rho\, \nabla \Phi_{\mathrm{PPC}} = \nabla P_g.}$$

With

$$P_g = \omega \rho c^2,$$

this gives

$$\boxed{\nabla P_g = c^2 \nabla(\omega \rho).}$$

Thus,

$$\boxed{\rho\, \mathbf{g} = -c^2 \nabla(\omega \rho).}$$

This is the **static weak-field fluid-force sector** of PPC.

---

8. Complete Tensor-to-Weak-Field Chain

The complete derivation can therefore be represented as

$$\boxed{T_{\mu\nu}^{\mathrm{PPC}} = (E_d + P_g)u_\mu u_\nu + P_g g_{\mu\nu}}$$

↓

$$\boxed{E_d = \rho c^2, \qquad P_g = \omega E_d}$$

↓

$$\boxed{R_{\mu\nu} = \frac{8\pi G}{c^4}\left(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T\right)}$$

↓

$$\boxed{\nabla^2 \Phi_{\mathrm{PPC}} = \frac{4\pi G}{c^2}(E_d + 3P_g)}$$

↓

$$\boxed{g^{\mathrm{PPC}}_{\mu\nu} \simeq \eta_{\mu\nu} + h_{\mu\nu}(\Phi_{\mathrm{PPC}})}$$

↓

$$\boxed{\Gamma^{\mu}_{\alpha\beta}[g_{\mathrm{PPC}}]}$$

↓

$$\boxed{\text{geodesic motion.}}$$

Separately, conservation of the perfect-fluid tensor gives

$$\boxed{\nabla_\mu T^{\mu\nu}_{\mathrm{PPC}} = 0}$$

↓

$$\boxed{(E_d + P_g)a_\mu = -h_\mu{}^\nu \nabla_\nu P_g}$$

↓

$$\boxed{\rho \mathbf{g} = -\nabla P_g}$$

in the Newtonian/static limit.

Key point

So there are **two related but distinct weak-field equations**:

Active gravitational curvature:

$$\boxed{\nabla^2 \Phi_{\mathrm{PPC}} = \frac{4\pi G}{c^2}(E_d + 3P_g)}$$

and

Static fluid-force balance:

$$\rho \mathbf{g} = -\nabla P_g.$$

That distinction is important for [illegible] PRC paper. The first connects the **perfect-fluid tensor to spacetime curvature**, while the second connects **pressure gradients to fluid acceleration**.

The two equations we need to keep separate are

$$\boxed{\nabla^2 \Phi_{\mathrm{PPC}} = \frac{4\pi G}{c^2}(E_d + 3P_g)}$$

and the static fluid-force relation

$$\boxed{\rho\,\mathbf{g} = -\nabla P_g, \qquad \mathbf{g} = -\nabla \Phi_{\mathrm{PPC}}.}$$

With

$$E_d = \rho c^2, \qquad P_g = \omega E_d = \omega \rho c^2.$$

Case 1 — Strict Vacuum

Take

$$\boxed{E_d = 0}$$

and therefore

$$\boxed{P_g = 0}.$$

Then

$$E_d + 3P_g = 0,$$

so the weak-field source equation becomes

$$\boxed{\nabla^2 \Phi_{\mathrm{PPC}} = 0.}$$

Also,

$$\boxed{\nabla P_g = 0.}$$

Thus the pressure-force equation gives no local pressure force:

$$\boxed{\rho \mathbf{g} = -\nabla P_g = 0.}$$

But this **does not mean that the solar gravitational field disappears**. The vacuum equation admits the exterior solution

$$\boxed{\Phi(r) = -\frac{GM_\odot}{r}}$$

with $M_\odot$ appearing as the integration constant determined by the central gravitating body.

with $M_\odot$ appearing as the integration constant determined by the central gravitating body.

The corresponding nonlinear vacuum solution is Schwarzschild:

$$\boxed{g_{\mu\nu}^{\mathrm{PPC}} = g_{\mu\nu}^{\mathrm{Schwarzschild}}.}$$

Therefore

$$\boxed{\Delta\phi_{\mathrm{PPC}} = 42.98''/\mathrm{century}}$$

for Mercury.

Result

$$\boxed{\textbf{Case 1: PPC} = \textbf{GR vacuum prediction.}}$$

Case 2 — Very Small E_d, Very Small P_g

Now take

$$\boxed{0 < E_d \ll \text{solar gravitational energy scale}}$$

and

$$\boxed{\omega \ll 1.}$$

Therefore

$$P_g = \omega E_d$$

is extremely small:

$$\boxed{P_g \ll E_d.}$$

The active source becomes

$$E_d + 3P_g = E_d(1 + 3\omega).$$

Hence

$$\boxed{\nabla^2 \Phi_{\mathrm{PPC}} = \frac{4\pi G}{c^2} E_d(1 + 3\omega).}$$

Since both E_d and ω are very small, the additional PPC contribution is very small.

Define schematically

$$\boxed{\beta_{\mathrm{PPC}} \propto \frac{P_g}{E_d} \sim \omega.}$$

Then

$$\boxed{\beta_{\mathrm{PPC}} \to 0.}$$

Consequently,

$$\boxed{g_{\mu\nu}^{\mathrm{PPC}} \to g_{\mu\nu}^{\mathrm{Schwarzschild}}}$$

and

PPC CONNECTION REDUCES TO SCHWARZSCHILD CONNECTION

$$\Gamma^{\mu}_{\alpha\beta}{}^{\mathrm{PPC}} \longrightarrow \Gamma^{\mu}_{\alpha\beta}{}^{\mathrm{Schwarzschild}}$$

Meaning: When the PPC fields take the Schwarzschild vacuum values ($P_g = 0$, $E_d = \rho c^2$ with $\rho = 0$ outside the source), the PPC metric becomes the Schwarzschild metric, and therefore the PPC Christoffel symbols $\Gamma^{\mu\,\mathrm{PPC}}_{\alpha\beta}$ reduce exactly to the Schwarzschild Christoffel symbols $\Gamma^{\mu}_{\alpha\beta}{}^{\mathrm{Schwarzschild}}$.

Mathematical Statement: If $g_{\mu\nu}^{\mathrm{PPC}} \to g_{\mu\nu}^{\mathrm{Schwarzschild}}$, then $\Gamma^{\mu}_{\alpha\beta}{}^{\mathrm{PPC}} \to \Gamma^{\mu}_{\alpha\beta}{}^{\mathrm{Schwarzschild}}$.

Physical Interpretation: The PPC connection reproduces the Schwarzschild connection in the appropriate vacuum limit.
$\Longrightarrow$ PPC is consistent with General Relativity in the Schwarzschild regime.

The perihelion prediction therefore approaches the GR value:

$$\boxed{\Delta\phi_{\mathrm{PPC}} \to 42.98''/\mathrm{century}.}$$

The pressure-force contribution is simultaneously small because

$$\boxed{\nabla P_g = c^2 \nabla(\omega\rho).}$$

If both the density and pressure are very small and slowly varying, then

$$\boxed{|\nabla P_g| \ll 1}$$

in the corresponding physical units.

$$\boxed{\Delta\phi_{\rm PPC} = 42.98''/\text{century} + \delta\phi_{\rm PPC}, \qquad |\delta\phi_{\rm PPC}| \ll 42.98''/\text{century}.}$$

$$\boxed{\Delta\phi_{\rm PPC} \simeq 42.98''/\text{century}}$$

Case 3 — Near-Vacuum Limit

This is slightly different conceptually from Case 1.

Take

$$\boxed{E_d \to 0, \qquad P_g \to 0, \qquad \nabla P_g \to 0.}$$

Then

$$E_d + 3P_g \to 0$$

and therefore

$$\boxed{\nabla^2 \Phi_{\mathrm{PPC}} \to 0.}$$

The metric approaches the vacuum solution:

$$g_{\mu\nu}^{\mathrm{PPC}} \to g_{\mu\nu}^{\mathrm{Schwarzschild}}.$$

Consequently,

PPC CONNECTION REDUCES TO SCHWARZSCHILD CONNECTION

$$\Gamma^{\mu}_{\alpha\beta}{}^{\mathrm{PPC}} \longrightarrow \Gamma^{\mu}_{\alpha\beta}{}^{\mathrm{Schwarzschild}}.$$

Meaning: When the PPC fields take the Schwarzschild vacuum values ($P_g = 0$, $E_d = \rho c^2$ with $\rho = 0$ outside the source), the PPC metric becomes the Schwarzschild metric, and therefore the PPC Christoffel symbols $\Gamma^{\mu\,\mathrm{PPC}}_{\alpha\beta}$ reduce exactly to the Schwarzschild Christoffel symbols $\Gamma^{\mu}_{\alpha\beta}{}^{\mathrm{Schwarzschild}}$.

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Physical Interpretation: The PPC connection reproduces the Schwarzschild connection in the appropriate vacuum limit.
$\Rightarrow$ PPC is consistent with General Relativity in the Schwarzschild regime.

The orbital equation therefore approaches

$$\boxed{u'' + u = \frac{GM_\odot}{h^2} + \frac{3GM_\odot}{c^2}u^2.}$$

Hence

$$\boxed{\Delta\phi_{\mathrm{PPC}} \to 42.98''/\mathrm{century}.}$$

Case	E_d	P_g	∇P_g
1. Strict vacuum	0	0	0
2. Tiny source	$\ll 1$	$\ll 1$	tiny
3. Near vacuum	$\to 0$	$\to 0$	$\to 0$

The complete mathematical chain is

$$\boxed{E_d,\ P_g \to T^{\mathrm{PPC}}_{\mu\nu} \to G_{\mu\nu} \to \Phi_{\mathrm{PPC}} \to g^{\mathrm{PPC}}_{\mu\nu} \to \Gamma^{\mu}_{\alpha\beta} \to \text{geodesic} \to \Delta\phi_{\mathrm{PPC}}.}$$

The key conclusion

Case 1:	$E_d = P_g = 0$	$\Rightarrow$	$\Delta\phi = 42.98''/\text{century},$
Case 2:	E_d, P_g very small	$\Rightarrow$	$\Delta\phi_{\text{PPC}} \simeq 42.98''/\text{century},$
Case 3:	$E_d, P_g \to 0$	$\Rightarrow$	$\Delta\phi_{\text{PPC}} \to 42.98''/\text{century}.$

[8.] Separate Note — Static Weak-Field Limit from the PPC Perfect-Fluid Tensor

1. Fundamental PPC Perfect-Fluid Tensor

The PPC energy-momentum tensor is

\[\boxed{ T_{\mu\nu}^{\rm PPC} = (E_d+P_g)u_\mu u_\nu + P_g g_{\mu\nu}. } \]

The PPC source variables are

\[\boxed{E_d=\rho c^2} \]

and

\[\boxed{P_g=\omega E_d.} \]

Therefore,

\[\boxed{P_g=\omega\rho c^2.} \]

This tensor is the starting point for the static weak-field formulation.

1. Fundamental PPC Perfect-Fluid Tensor

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The PPC source variables are

$$\boxed{E_d = \rho c^2}$$

and

$$\boxed{P_g = \omega E_d.}$$

Therefore,

$$\boxed{P_g = \omega \rho c^2.}$$

This tensor is the starting point for the static weak-field formulation.

---

2. Static Fluid Limit

For a static fluid, the spatial components of the four-velocity vanish in the comoving frame. In that frame, the perfect-fluid tensor has the schematic form

\[\boxed{ T_{\mu\nu}^{\rm PPC} \sim \begin{pmatrix} E_d&0&0&0\\ 0&P_g&0&0\\ 0&0&P_g&0\\ 0&0&0&P_g \end{pmatrix}. } \]

Thus the source contains both the energy density \(E_d\) and the three isotropic pressure components.

With the metric signature \((-+++)\), its trace is

\[\boxed{ T=-E_d+3P_g. } \]

2. Static Fluid Limit

For a static fluid, the spatial components of the four-velocity vanish in the comoving frame. In that frame, the perfect-fluid tensor has the schematic form

$$\boxed{T^{\mathrm{PPC}}_{\mu\nu} \sim \begin{pmatrix} E_d & 0 & 0 & 0 \\ 0 & P_g & 0 & 0 \\ 0 & 0 & P_g & 0 \\ 0 & 0 & 0 & P_g \end{pmatrix}.}$$

Thus the source contains both the energy density E_d and the three isotropic pressure components.

With the metric signature $(-+++)$, its trace is

$$\boxed{T = -E_d + 3P_g.}$$

3. PPC Tensor Field Equation

The PPC tensor field equation is

$$\boxed{ G_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}^{\rm PPC}. }$$

Equivalently,

$$\boxed{ R_{\mu\nu} = \frac{8\pi G}{c^4} \left(T_{\mu\nu}^{\rm PPC} -\frac12g_{\mu\nu}T^{\rm PPC} \right). }$$

For the static weak-field gravitational potential, the energy density and the three pressure components combine into the active source

$$\boxed{ E_d+3P_g. }$$

Consequently, the PPC static weak-field equation is

$$\boxed{ \nabla^2\Phi_{\rm PPC} = \frac{4\pi G}{c^2} (E_d+3P_g). } \tag{A}$$

Using

$$E_d=\rho c^2, \qquad P_g=\omega E_d,$$

gives

$$\boxed{ \nabla^2\Phi_{\rm PPC} = 4\pi G\rho(1+3\omega). } \tag{B}$$

This is the **static weak-field curvature equation** of the PPC formulation.

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$$\boxed{R_{\mu\nu} = \frac{8\pi G}{c^4}\left(T^{\mathrm{PPC}}_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T^{\mathrm{PPC}}\right).}$$

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Equivalently,

$$R_{\mu\nu} = \frac{8\pi G}{c^4}\left(T^{\mathrm{PPC}}_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T^{\mathrm{PPC}}\right)$$

[A]

Using

$$E_d = \rho c^2, \qquad P_g = \omega E_d,$$

gives

$\nabla^2\Phi_{PPC} = 4\pi G\rho(1 + 3\omega)$ [B]

This is the **static weak-field curvature equation** of the PPC formulation.

---

4. Weak-Field Metric Tensor

The weak-field metric is written as

$$\boxed{ g_{\mu\nu}^{\rm PPC} = \eta_{\mu\nu}+h_{\mu\nu}, \qquad |h_{\mu\nu}|\ll 1. }$$

Using the PPC potential,

$$\boxed{ g_{00}^{\rm PPC} \simeq -\left(1+\frac{2\Phi_{\rm PPC}}{c^2}\right). }$$

For the chosen isotropic weak-field representation,

$$\boxed{ g_{ij}^{\rm PPC} \simeq \left(1-\frac{2\Phi_{\rm PPC}}{c^2}\right)\delta_{ij}. }$$

Therefore,

$$\boxed{ ds^2\simeq -\left(1+\frac{2\Phi_{\rm PPC}}{c^2}\right)c^2dt^2 + \left(1-\frac{2\Phi_{\rm PPC}}{c^2}\right) (dx^2+dy^2+dz^2). } \tag{C}$$

Thus,

$$\boxed{ E_d,P_g \rightarrow \Phi_{\rm PPC} \rightarrow g_{\mu\nu}^{\rm PPC}. }$$

4. Weak-Field Metric Tensor

The weak-field metric is written as

$$\boxed{g_{\mu\nu}^{\mathrm{PPC}} = \eta_{\mu\nu} + h_{\mu\nu}, \qquad |h_{\mu\nu}| \ll 1.}$$

Using the PPC potential,

$$\boxed{g_{00}^{\mathrm{PPC}} \simeq -\left(1 + \frac{2\Phi_{\mathrm{PPC}}}{c^2}\right).}$$

For the chosen isotropic weak-field representation,

$$\boxed{g_{ij}^{\mathrm{PPC}} \simeq \left(1 - \frac{2\Phi_{\mathrm{PPC}}}{c^2}\right)\delta_{ij}.}$$

Therefore,

$ds^2 \simeq -(1 + 2\Phi_{PPC}/c^2)c^2dt^2 + (1 - 2\Phi_{PPC}/c^2)(dx^2 + dy^2 + dz^2)$ [C]

Thus,

$$\boxed{E_d, P_g \rightarrow \Phi_{\rm PPC} \rightarrow g^{\rm PPC}_{\mu\nu}.}$$

---

5. Pressure-Gradient Equation

The perfect-fluid conservation equation is

$$\boxed{ \nabla_\mu T^{\mu\nu}_{\rm PPC}=0. }$$

Its spatial projection gives the relativistic fluid-force relation

$$\boxed{ (E_d+P_g)a_\mu = -h_\mu{}^\nu\nabla_\nu P_g. } \tag{D}$$

In the weak-field, nonrelativistic limit, this becomes the PPC pressure-force relation

$$\boxed{ \rho\,\mathbf g=-\nabla P_g. } \tag{E}$$

Since

$$\mathbf g=-\nabla\Phi_{\rm PPC},$$

we have

$$\boxed{ \rho\,\nabla\Phi_{\rm PPC} = \nabla P_g. }$$

And because

\[P_g=\omega\rho c^2, \] \[\boxed{ \nabla P_g = c^2\nabla(\omega\rho). } \]

Thus the pressure-gradient sector is

\[\boxed{ \rho\mathbf g = -c^2\nabla(\omega\rho). } \tag{F} \]

5. Pressure-Gradient Equation

The perfect-fluid conservation equation is

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In the weak-field, nonrelativistic limit, this becomes the PPC pressure-force relation

$\rho \vec{g} = -\nabla P\_g$ [E]

Since

$$\mathbf{g} = -\nabla\Phi_{\mathrm{PPC}},$$

we have

$$\boxed{\rho\,\nabla\Phi_{\mathrm{PPC}} = \nabla P_g.}$$

And because

$$P_g = \omega\rho c^2,$$

$$\boxed{\nabla P_g = c^2\nabla(\omega\rho).}$$

Thus the pressure-gradient sector is

$$\rho\,\vec{g} = -c^2\,\nabla(\omega\rho) \quad [F]$$

---

6. Case 1 — Exact Vacuum / Claim A

Take

$$\boxed{ E_d=0. }$$

Then

$$\boxed{ P_g=\omega E_d=0. }$$

Consequently,

$$\boxed{ E_d+3P_g=0 }$$

and equation (A) gives

$$\boxed{ \nabla^2\Phi_{\rm PPC}=0. }$$

Also,

$$\boxed{ \nabla P_g=0. }$$

The full tensor equation becomes

$$\boxed{ G_{\mu\nu}=0. }$$

For a static, spherical, asymptotically flat exterior, the full nonlinear solution is the Schwarzschild metric.

Therefore,

$$\boxed{ g_{\mu\nu}^{\rm PPC} = g_{\mu\nu}^{\rm Schwarzschild}. }$$

The Mercury geodesic then gives

$$\boxed{ \Delta\phi \simeq42.98''/\text{century}. }$$

Interpretation

Claim A is an exact vacuum recovery of the GR exterior prediction.

6. Case 1 — Exact Vacuum / Claim A

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For a static, spherical, asymptotically flat exterior, the full nonlinear solution is the Schwarzschild metric.

Therefore,

$$\boxed{g_{\mu\nu}^{\mathrm{PPC}} = g_{\mu\nu}^{\mathrm{Schwarzschild}}.}$$

The Mercury geodesic then gives

$$\boxed{\Delta\phi \simeq 42.98''/\mathrm{century}.}$$

7. Case 2 — Very Small Nonzero PPC Source

Now consider

$$\boxed{ E_d>0, \qquad E_d\ll\text{ordinary gravitational energy scale}, }$$

with

$$\boxed{ \omega\ll1. }$$

Then

$$P_g=\omega E_d$$

is extremely small:

$$\boxed{ P_g\ll E_d. }$$

The weak-field source becomes

$$\boxed{ \nabla^2\Phi_{\rm PPC} = \frac{4\pi G}{c^2} E_d(1+3\omega). }$$

Thus the pressure contribution

$$3P_g=3\omega E_d$$

is very small.

If the spatial variation is also weak,

$$\boxed{ |\nabla P_g|\approx0, }$$

then the pressure-force contribution

$$\boxed{ \rho\mathbf g=-\nabla P_g }$$

is correspondingly small.

Therefore the PPC metric is expected to be close to the Schwarzschild metric:

$$\boxed{ g_{\mu\nu}^{\rm PPC} \simeq g_{\mu\nu}^{\rm Schwarzschild}. }$$

Consequently, the PPC connection approaches the Schwarzschild connection:

$$\boxed{ \Gamma^\mu_{\alpha\beta}^{\rm PPC} \simeq \Gamma^\mu_{\alpha\beta}^{\rm Schwarzschild}. }$$

Parameterizing the residual orbital correction as

$$\boxed{ u''+u = \frac{GM_\odot}{h^2} + (3+\beta_{\rm PPC}) \frac{GM_\odot}{c^2}u^2, }$$

the intended weak-source limit is

$$\boxed{ \beta_{\rm PPC}\approx0. }$$

Therefore,

$$\boxed{ \Delta\phi_{\rm PPC} \simeq42.98''/\text{century}. }$$

Important qualification

The statement

$$\boxed{\beta_{\rm PPC}\approx0}$$

must ultimately be **derived from the explicit PPC metric**, rather than assumed solely because E_d and P_g are small.

If a detailed calculation gives a value such as

$$\boxed{ \Delta\phi_{\rm PPC}=42.9987''/\text{century}, }$$

that value should be reported as the **specific PPC calculation**, with the corresponding E_d,ω, metric, and orbital solution stated explicitly.

Interpretation

Claim A is an exact vacuum recovery of the GR exterior prediction.

---

7. Case 2 — Very Small Nonzero PPC Source

Now consider

$$\boxed{E_d > 0, \qquad E_d \ll \text{ordinary gravitational energy scale},}$$

with

$$\boxed{\omega \ll 1.}$$

Then

$$P_d = \omega E_d$$

Then

$$P_g = \omega E_d$$

is extremely small:

$$\boxed{P_g \ll E_d.}$$

The weak-field source becomes

$$\boxed{\nabla^2 \Phi_{\mathrm{PPC}} = \frac{4\pi G}{c^2} E_d (1 + 3\omega).}$$

Thus the pressure contribution

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Consequently, the PPC connection approaches the Schwarzschild connection:

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$$\boxed{u'' + u = \frac{GM_\odot}{h^2} + (3 + \beta_{\mathrm{PPC}})\frac{GM_\odot}{c^2}u^2,}$$

the intended weak-source limit is

$$\boxed{\beta_{\mathrm{PPC}} \approx 0.}$$

Therefore,

$$\boxed{\Delta\phi_{\mathrm{PPC}} \simeq 42.98''/\mathrm{century}.}$$

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If a detailed calculation gives a value such as

$$\boxed{\Delta\phi_{\mathrm{PPC}} = 42.9987'' / \mathrm{century},}$$

that value should be reported as the **specific PPC calculation**, with the corresponding E_d, ω, metric, and orbital solution stated explicitly.

8. Case 3 — Near-Vacuum Limit

Case 3 describes the continuous limit

$$\boxed{ E_d\rightarrow0, \qquad P_g\rightarrow0, \qquad |\nabla P_g|\rightarrow0. }$$

Since

$$E_d+3P_g\rightarrow0,$$

the weak-field equation gives

$$\boxed{ \nabla^2\Phi_{\rm PPC}\rightarrow0. }$$

The PPC metric therefore approaches the vacuum metric:

$$\boxed{ g_{\mu\nu}^{\rm PPC} \rightarrow g_{\mu\nu}^{\rm Schwarzschild}. }$$

Similarly,

$$\boxed{ \Gamma^\mu_{\alpha\beta}^{\rm PPC} \rightarrow \Gamma^\mu_{\alpha\beta}^{\rm Schwarzschild}. }$$

The orbital correction satisfies

$$\boxed{ \beta_{\rm PPC}\rightarrow0, }$$

provided the full PPC field equations possess a regular continuous vacuum limit.

Therefore,

$$\boxed{ \Delta\phi_{\rm PPC} \rightarrow 42.98''/\text{century}. }$$

Interpretation

Case 3 is not exact vacuum. It is the **mathematical continuity limit toward vacuum**.

8. Case 3 — Near-Vacuum Limit

Case 3 describes the continuous limit

$$\boxed{E_d \to 0, \qquad P_g \to 0, \qquad |\nabla P_g| \to 0.}$$

Since

$$E_d + 3P_g \to 0,$$

the weak-field equation gives

$$\boxed{\nabla^2 \Phi_{\mathrm{PPC}} \to 0.}$$

The PPC metric therefore approaches the vacuum metric:

$$\boxed{g_{\mu\nu}^{\mathrm{PPC}} \to g_{\mu\nu}^{\mathrm{Schwarzschild}}.}$$

The orbital correction satisfies

$$\boxed{\beta_{\mathrm{PPC}} \to 0,}$$

provided the full PPC field equations possess a regular continuous vacuum limit.

Therefore,

$$\boxed{\Delta\phi_{\mathrm{PPC}} \to 42.98''/\mathrm{century}.}$$

Interpretation

Case 3 is not exact vacuum. It is the **mathematical continuity limit toward vacuum**.

9. Comparison of the Three Cases

PPC Conditions and Perihelion Advance of Mercury

Case	E_d	P_g	|∇P_g|	Result
Case 1 — Claim A	= 0	= 0	= 0	Exact Schwarzschild → **42.98″/century**
Case 2 — Weak PPC source	Very small, nonzero	Very small	Very small	β_PPC ≈ 0 → approximately **42.98″/century**
Case 3 — Near-vacuum limit	→ 0	→ 0	→ 0	Schwarzschild limit → **42.98″/century**

Summary

In all three cases, as **(E_d, P_g, |∇P_g|) → (0, 0, 0)**, the PPC solution approaches the Schwarzschild solution, and the perihelion advance of Mercury approaches the general-relativistic value of **42.98″ per century**.

PPC CONDITIONS AND PERIHELION ADVANCE OF MERCURY

Case	E_d	P_g	$\lvert\nabla P_g\rvert$	Result
Case 1 — Claim A	$= 0$	$= 0$	$= 0$	Exact Schwarzschild → $42.98''$/century
Case 2 — Weak PPC source	Very small, nonzero	Very small (nonzero)	Very small (nonzero)	$\beta_{\mathrm{PPC}} \approx 0$ → approximately $42.98''$/century
Case 3 — Near-vacuum limit	$\to 0$	$\to 0$	$\to 0$	Schwarzschild limit → $42.98''$/century

Summary: In all three cases, as $(E_d, P_g, |\nabla P_g|) \to (0,0,0)$, the PPC solution approaches the Schwarzschild solution and the perihelion advance of Mercury approaches the general relativistic value of $42.98''$ per century.

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10. Complete Tensor-to-Observable Chain

The complete PPC structure is

\[\boxed{ T_{\mu\nu}^{\rm PPC} = (E_d+P_g)u_\mu u_\nu + P_g g_{\mu\nu} } \]

↓

\[\boxed{ E_d=\rho c^2, \qquad P_g=\omega E_d } \]

↓

$$\boxed{ \nabla^2\Phi_{\rm PPC} = \frac{4\pi G}{c^2}(E_d+3P_g) }$$

↓

$$\boxed{ g_{\mu\nu}^{\rm PPC} \simeq \eta_{\mu\nu}+h_{\mu\nu}(\Phi_{\rm PPC}) }$$

↓

$$\boxed{ g^{\mu\nu}_{\rm PPC} }$$

↓

$$\boxed{ \Gamma^\mu_{\alpha\beta} [g_{\rm PPC}] }$$

↓

$$\boxed{ \frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu_{\alpha\beta} \frac{dx^\alpha}{d\tau} \frac{dx^\beta}{d\tau} =0 }$$

↓

$$\boxed{ \text{planetary orbit} }$$

↓

$$\boxed{ \Delta\phi_{\rm PPC}. }$$

The separate pressure-force branch is

$$\boxed{ \nabla_\mu T^{\mu\nu}_{\rm PPC}=0 }$$

↓

$$\boxed{ (E_d+P_g)a_\mu = -h_\mu{}^\nu\nabla_\nu P_g }$$

↓

$$\boxed{ \rho\mathbf g=-\nabla P_g }$$

in the Newtonian/static limit.

10. Complete Tensor-to-Observable Chain

The complete PPC structure is

$$\boxed{T_{\mu\nu}^{\mathrm{PPC}} = (E_d + P_g)u_\mu u_\nu + P_g g_{\mu\nu}}$$

↓

$$\boxed{E_d = \rho c^2, \qquad P_g = \omega E_d}$$

↓

$$\boxed{\nabla^2 \Phi_{\mathrm{PPC}} = \frac{4\pi G}{c^2}(E_d + 3P_g)}$$

↓

$$\boxed{g_{\mu\nu}^{\mathrm{PPC}} \simeq \eta_{\mu\nu} + h_{\mu\nu}(\Phi_{\mathrm{PPC}})}$$

↓

$$\boxed{g_{\mathrm{PPC}}^{\mu\nu}}$$

↓

$$\boxed{\Gamma_{\alpha\beta}^{\mu}[g_{\mathrm{PPC}}]}$$

↓

$$\boxed{\frac{d^2x^\mu}{d\tau^2} + \Gamma_{\alpha\beta}^{\mu}\frac{dx^\alpha}{d\tau}\frac{dx^\beta}{d\tau} = 0}$$

↓

$$\boxed{\text{planetary orbit}}$$

↓

$$\boxed{\Delta\phi_{\mathrm{PPC}}.}$$

The separate pressure-force branch is

$$\boxed{\nabla_\mu T^{\mu\nu}_{\mathrm{PPC}} = 0}$$

↓

$$\boxed{(E_d + P_g)a_\mu = -h_\mu{}^\nu \nabla_\nu P_g}$$

↓

$$\boxed{\rho \mathbf{g} = -\nabla P_g}$$

in the Newtonian/static limit.

This is the formulation I would use in the paper: the perfect-fluid tensor supplies the covariant source; its static weak-field limit gives the (E_d+3P_g) gravitational source; the pressure gradient separately supplies the fluid-force balance; and the metric then determines the connection and geodesic observable.

$$R_{\mu\nu} = \frac{8\pi G}{c^4}\left(T^{\mathrm{PPC}}_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T^{\mathrm{PPC}}\right)$$

Reversed Trace Equation — PPC Perfect Fluid

Starting from

$$\boxed{G_{\mu\nu} = \frac{8\pi G}{c^4} T^{\mathrm{PPC}}_{\mu\nu}}$$

take the trace:

$$G^{\mu}{}_{\mu} = -R$$

and therefore

$$\boxed{-R = \frac{8\pi G}{c^4} T^{\mathrm{PPC}}.}$$

Hence the **reversed trace relation** is

$$\boxed{T^{\mathrm{PPC}} = -\frac{c^4}{8\pi G} R.}$$

For the perfect fluid,

$$\boxed{T^{\mathrm{PPC}} = -E_d + 3P_g.}$$

Therefore,

$$-E_d + 3P_g = -\frac{c^4}{8\pi G} R.$$

Multiplying by -1,

$E_d - 3P_g = (c^4 / 8\pi G)\, R$ [RT1]

Thus the Ricci scalar can equivalently be written as

$R = (8\pi G/c^4)(E_d - 3P_g)$ [RT2]

Using

$$P_g = \omega E_d,$$

we obtain

$$\boxed{R = \frac{8\pi G}{c^4} E_d (1 - 3\omega).}$$

And using $E_d = \rho c^2$,

$$\boxed{R = \frac{8\pi G}{c^2} \rho (1 - 3\omega).}$$

Therefore the reversed chain is

$$\boxed{R \longrightarrow E_d - 3P_g \longrightarrow T^{\mathrm{PPC}}}$$

with

$$\boxed{E_d - 3P_g = \frac{c^4}{8\pi G} R.}$$

For your three cases:

Case 1:

$$E_d = P_g = 0 \quad \Rightarrow \quad \boxed{R = 0}.$$

Case 2:

$$E_d \neq 0, \quad P_g \ll E_d \quad \Rightarrow \quad \boxed{R \approx \frac{8\pi G}{c^4} E_d}$$

and R is very small.

Case 3:

$$E_d \to 0, \quad P_g \to 0 \quad \Rightarrow \quad \boxed{R \to 0}.$$

This is the cleanest **reverse-trace form** for
PPC tensor formulation.

$$\boxed{T_{\mu\nu}^{\mathrm{PPC}} = (E_d + P_g)u_\mu u_\nu + P_g g_{\mu\nu}.}$$

Its trace is

$$\boxed{T^{\mathrm{PPC}} = g^{\mu\nu} T_{\mu\nu}^{\mathrm{PPC}} = -E_d + 3P_g.}$$

The PPC field equation is

$$\boxed{G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}^{\mathrm{PPC}}.}$$

Taking the trace,

$$-R = \tfrac{8\pi G}{c^4} T^{\mathrm{PPC}}.$$

Therefore,

$$\boxed{R = -\frac{8\pi G}{c^4} T^{\mathrm{PPC}}.}$$

Three Cases

Case 1 — Exact Vacuum

$$E_d = 0, \qquad P_g = 0$$

therefore

$$\boxed{R = 0.}$$

This is consistent with the Schwarzschild exterior:

$$\boxed{R_{\text{Schwarzschild}} = 0.}$$

Importantly, $R = 0$ does **not** mean the entire curvature tensor is zero; the Schwarzschild spacetime has nonzero Ricci/Weyl curvature structure, with the Ricci tensor vanishing in vacuum.

Case 2 — Very Small PPC Source

$$E_d \neq 0, \qquad E_d \ll 1,$$

$$\omega \ll 1, \qquad P_g = \omega E_d \approx 0.$$

Then

$$\boxed{R = \frac{8\pi G}{c^4}(E_d - 3P_g) \approx \frac{8\pi G}{c^4} E_d.}$$

Hence

$$\boxed{R \approx 0}$$

but it is **not exactly zero**.

$$\boxed{\beta_{\mathrm{PPC}} \approx 0}$$

only if the explicit PPC metric calculation confirms that the resulting curvature correction to the Mercury orbit is negligible.

Case 3 — Near-Vacuum Limit

$$E_d \to 0, \qquad P_g \to 0.$$

Then

$$\boxed{R \to 0.}$$

Consequently,

$$\boxed{g_{\mu\nu}^{\mathrm{PPC}} \to g_{\mu\nu}^{\mathrm{Schwarzschild}}}$$

and

$$\boxed{\Delta\phi_{\mathrm{PPC}} \to 42.98''/\mathrm{century}.}$$

$$T^{\mathrm{PPC}}_{\mu\nu} \to T^{\mathrm{PPC}} \to R_{\mu\nu}, R \to G_{\mu\nu} \to g^{\mathrm{PPC}}_{\mu\nu} \to \Gamma^{\mu}_{\alpha\beta} \to \text{geodesic} \to \Delta\phi_{\mathrm{PPC}}.$$

with the key scalar relation

$$R = \frac{8\pi G}{c^4}(E_d - 3P_g) = \frac{8\pi G}{c^4} E_d(1 - 3\omega).$$

Numerical Calculation of Mercury's Perihelion Advance

The first post-Newtonian orbital correction gives

$$\boxed{\Delta\phi = \frac{6\pi G M_\odot}{a(1-e^2)c^2}}$$

per Mercury revolution.

For Mercury, representative orbital parameters are

$$a = 0.3871\,\mathrm{AU}, \qquad e = 0.2056,$$

consistent with NASA orbital data.

NASA Technical Reports Server +1

Use

$$G = 6.67430 \times 10^{-11}\ \mathrm{m^3\,kg^{-1}\,s^{-2}},$$

$$M_\odot = 1.98847 \times 10^{30}\ \mathrm{kg},$$

$$c = 299\,792\,458\ \mathrm{m/s},$$

and

$$1\ \mathrm{AU} = 149\,597\,870\,700\ \mathrm{m}.$$

1. Convert Mercury's semimajor axis

$$a = 0.3871(149\,597\,870\,700)$$

so

$$\boxed{a \approx 5.789 \times 10^{10}\ \mathrm{m}.}$$

2. Calculate $1 - e^2$

$e = 0.2056$

therefore

$e^2 = 0.04227136$

and

$$\boxed{1 - e^2 = 0.95772864.}$$

3. Calculate $GM_\odot$

$$GM_\odot = (6.67430 \times 10^{-11})(1.98847 \times 10^{30})$$

giving

$$\boxed{GM_\odot \approx 1.327164 \times 10^{20}\ \mathrm{m}^3/\mathrm{s}^2.}$$

4. Calculate the relativistic angle per Mercury orbit

Insert the values:

$$\Delta\phi_{\text{orbit}} = \frac{6\pi(1.327164\times10^{20})}{(5.789\times10^{10})(0.95772864)(299792458)^2}\,.$$

This gives approximately

$$\boxed{\Delta\phi_{\text{orbit}} \approx 5.018 \times 10^{-7}\ \text{rad.}}$$

---

5. Convert radians to arcseconds

Using

$$1 \text{ rad} = 206264.806'',$$

we obtain

$$\Delta\phi_{\text{orbit}} = (5.018 \times 10^{-7})(206264.806'').$$

Therefore,

$$\boxed{\Delta\phi_{\text{orbit}} \approx 0.10352''.}$$

So Mercury advances by approximately

0.1035 arcseconds per orbit.

6. Number of Mercury orbits per century

Mercury's orbital period is approximately

$$T = 87.9691 \text{ days}.$$

A Julian century contains

$$36525 \text{ days}.$$

Therefore,

$$N = \frac{36525}{87.9691}$$

giving

$$\boxed{N \approx 415.20}$$

Mercury revolutions per century.

7. Multiply by the number of orbits

$$\Delta\phi_{\text{century}} = 0.10352'' \times 415.20.$$

Therefore,

$$\boxed{\Delta\phi_{\text{century}} \approx 42.981''/\text{century}.}$$

Rounded appropriately,

$$\boxed{\Delta\phi \approx 42.98''/\text{century}.}$$

This agrees with the standard quoted relativistic Mercury contribution of about $42.98''$ per century. NASA Technical Reports Server

$$\boxed{\Delta\phi_{\mathrm{PPC}} = \frac{6\pi G M_\odot}{a(1-e^2)c^2}\left(1+\frac{\beta_{\mathrm{PPC}}}{3}\right).}$$

The numerical GR-limit factor is

$$\frac{6\pi G M_\odot}{a(1-e^2)c^2} \times \frac{36525}{87.9691} \times 206264.806 \approx 42.981''/\mathrm{century}.$$

Thus

$$\boxed{\Delta\phi_{\mathrm{PPC}} \approx 42.981\left(1+\frac{\beta_{\mathrm{PPC}}}{3}\right)''/\mathrm{century}.}$$

Consequently, for your proposed weak-PPC case,

$$\boxed{\beta_{\mathrm{PPC}} \approx 0}$$

$$\boxed{\beta_{\mathrm{PPC}} \approx 0}$$

gives

$$\boxed{\Delta\phi_{\mathrm{PPC}} \approx 42.98''/\mathrm{century}.}$$

Important distinction:

This calculation establishes the Schwarzschild/GR limiting value. It does not yet establish a PPC-specific value such as 42.9987″/century.

NASA also notes that Mercury's total observed precession contains contributions from planetary perturbations and solar structure, with the relativistic contribution being one component.

Calculation Method — Mercury Perihelion Advance in PPC

The cleanest method is to calculate the perihelion advance **from the PPC metric**, rather than inserting the observed value beforehand.

1. Start from the PPC perfect-fluid source

$$\boxed{T_{\mu\nu}^{\mathrm{PPC}} = (E_d + P_g)u_\mu u_\nu + P_g g_{\mu\nu}}$$

with

$$\boxed{E_d = \rho c^2, \qquad P_g = \omega E_d.}$$

The PPC curvature scalar is

$$\boxed{R = \frac{8\pi G}{c^4}(E_d - 3P_g).}$$

2. Determine the PPC metric

The fundamental step is to obtain

$$\boxed{g_{\mu\nu}^{\mathrm{PPC}} = g_{\mu\nu}(E_d, P_g).}$$

For a static spherical field, use

$$\boxed{ds^2 = -e^{2\nu(r)}c^2dt^2 + e^{2\lambda(r)}dr^2 + r^2d\Omega^2.}$$

The functions $\nu(r)$ and $\lambda(r)$ must be obtained from the PPC field equations and the assumed $E_d(r)$, $P_g(r)$.

3. Calculate the inverse metric and connection

Obtain

$$\boxed{g^{\mu\nu}_{\mathrm{PPC}} = (g^{\mathrm{PPC}}_{\mu\nu})^{-1}.}$$

Then calculate

$$\boxed{\Gamma^{\mu}_{\alpha\beta} = \frac{1}{2} g^{\mu\nu} \left(\partial_\alpha g_{\nu\beta} + \partial_\beta g_{\nu\alpha} - \partial_\nu g_{\alpha\beta} \right).}$$

This is the point where the PPC source enters the orbital dynamics geometrically.

4. Derive the geodesic equation

For Mercury,

$$\boxed{\frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu_{\alpha\beta}\frac{dx^\alpha}{d\tau}\frac{dx^\beta}{d\tau} = 0.}$$

Because the metric is static and spherically symmetric, there are conserved quantities associated with time translation and rotational symmetry.

For equatorial motion,

$$\boxed{\theta = \frac{\pi}{2}.}$$

Define

$$\boxed{u = \frac{1}{r}.}$$

5. Obtain the PPC orbital equation

After eliminating the conserved energy and angular momentum, reduce the radial geodesic equation to an equation in $u(\phi)$.

Write the result as

$$\boxed{\frac{d^2u}{d\phi^2} + u = \frac{GM_\odot}{h^2} + \frac{3GM_\odot}{c^2}u^2 + \delta_{\mathrm{PPC}}(u).}$$

Here

$$\boxed{\delta_{\mathrm{PPC}}(u)}$$

is the correction produced specifically by the PPC metric.

This is the **decisive equation for Claim B**.

6. Define the PPC correction parameter

If the PPC correction has the same u^2 structure at first post-Newtonian order, write

$$\boxed{\delta_{\mathrm{PPC}}(u) = \beta_{\mathrm{PPC}} \frac{GM_\odot}{c^2} u^2.}$$

Then

$$\boxed{u'' + u = \frac{GM_\odot}{h^2} + (3 + \beta_{\mathrm{PPC}}) \frac{GM_\odot}{c^2} u^2.}$$

Important: β_{PPC} must be calculated from the explicit PPC metric. It should not be assigned a value merely because E_d or P_g is small.

7. Calculate the perihelion shift

For an equation of the form

$$u'' + u = \frac{GM_\odot}{h^2} + A\frac{GM_\odot}{c^2}u^2,$$

the first-order perihelion advance is

$$\boxed{\Delta\phi = \frac{2\pi A G M_\odot}{a(1-e^2)c^2}.}$$

For PPC,

$$A = 3 + \beta_{\mathrm{PPC}}.$$

Therefore,

$$\boxed{\Delta\phi_{\mathrm{PPC}} = \frac{2\pi(3+\beta_{\mathrm{PPC}})GM_\odot}{a(1-e^2)c^2}.}$$

Equivalently,

$$\boxed{\Delta\phi_{\mathrm{PPC}} = \frac{6\pi G M_\odot}{a(1-e^2)c^2}\left(1+\frac{\beta_{\mathrm{PPC}}}{3}\right).}$$

Thus,

$$\boxed{\Delta\phi_{\mathrm{PPC}} = \Delta\phi_{\mathrm{GR}}\left(1+\frac{\beta_{\mathrm{PPC}}}{3}\right).}$$

8. Mercury's GR limit

Using Mercury's orbital parameters gives the standard relativistic value

$$\boxed{\Delta\phi_{\mathrm{GR}} \approx 42.98'' / \mathrm{century}.}$$

Therefore,

$$\boxed{\Delta\phi_{\mathrm{PPC}} \approx 42.98 \left(1 + \frac{\beta_{\mathrm{PPC}}}{3}\right)'' / \mathrm{century}.}$$

If

$$\boxed{\beta_{\mathrm{PPC}} \approx 0,}$$

then

$$\boxed{\Delta\phi_{\mathrm{PPC}} \approx 42.98'' / \mathrm{century}.}$$

Three PPC Cases

Case 1 — Exact Vacuum

$$E_d = P_g = 0$$

$$\Downarrow$$

$$G_{\mu\nu} = 0$$

$$\Downarrow$$

Schwarzschild metric

$$\Downarrow$$

$$\boxed{\beta_{\mathrm{PPC}} = 0}$$

$$\Downarrow$$

$$\boxed{\Delta\phi = 42.98''/\mathrm{century}.}$$

Case 2 — Very Small PPC Source

$$E_d \neq 0, \qquad E_d \ll 1$$

$$\omega \ll 1$$

$$P_g = \omega E_d \approx 0$$

$$|\nabla P_g| \approx 0$$

$$\Downarrow$$

$$g_{\mu\nu}^{\mathrm{PPC}} \approx g_{\mu\nu}^{\mathrm{Schwarzschild}}$$

$$\Downarrow$$

$$\boxed{\beta_{\mathrm{PPC}} \approx 0}$$

$$\Downarrow$$

$$\boxed{\Delta\phi_{\mathrm{PPC}} \approx 42.98''/\mathrm{century}.}$$

Case 3 — Near-Vacuum Limit

$$E_d \to 0, \qquad P_g \to 0, \qquad \nabla P_g \to 0$$

$$\Downarrow$$

$$g_{\mu\nu}^{\mathrm{PPC}} \to g_{\mu\nu}^{\mathrm{Schwarzschild}}$$

$$\Downarrow$$

$$\beta_{\mathrm{PPC}} \to 0$$

$$\Downarrow$$

$$\boxed{\Delta\phi_{\mathrm{PPC}} \to 42.98''/\mathrm{century}.}$$

The calculation should therefore follow exactly this chain:

$$\boxed{E_d, P_g \to T_{\mu\nu}^{\mathrm{PPC}} \to G_{\mu\nu} \to g_{\mu\nu}^{\mathrm{PPC}} \to g_{\mathrm{PPC}}^{\mu\nu} \to \Gamma_{\alpha\beta}^{\mu} \to \text{geodesic} \to u(\phi) \to \beta_{\mathrm{PPC}} \to \Delta\phi_{\mathrm{PPC}}.}$$

Separate Note — Mercury Perihelion: Observed Components and PPC Contribution

1. Total Observed Perihelion Advance

The observed secular motion of Mercury's perihelion contains several physical contributions. It should therefore not be identified directly with the $42.98''/\text{century}$ relativistic term.

The total perihelion motion may be schematically represented as

$$\Delta\phi_{\text{total}} = \Delta\phi_{\text{planetary}} + \Delta\phi_{\text{solar}} + \Delta\phi_{\text{relativistic}} + \cdots$$

The planetary perturbations constitute the dominant contribution, while the relativistic contribution from the Sun is approximately

$$\boxed{\Delta\phi_{\mathrm{GR}} \simeq 42.98''/\mathrm{century}.}$$

The solar contribution includes smaller effects associated with the Sun's physical properties, such as its quadrupole moment and angular momentum.

2. Relativistic Reference Value

For a test body moving in the Schwarzschild exterior,

$$\boxed{\Delta\phi_{\mathrm{GR}} = \frac{6\pi G M_\odot}{a(1-e^2)c^2}.}$$

For Mercury, this gives approximately

$$\boxed{\Delta\phi_{\mathrm{GR}} \simeq 42.981''/\mathrm{century}.}$$

For observational discussion this is conventionally rounded to

$$\boxed{\Delta\phi_{\mathrm{GR}} \simeq 43''/\mathrm{century}.}$$

This $43''/\mathrm{century}$ quantity refers to the **relativistic/anomalous component**, not the entire observed perihelion motion.

3. PPC Reference

The PPC calculation should therefore compare its prediction with the relativistic reference:

$$\boxed{\Delta\phi_{\mathrm{PPC}} \quad \text{vs.} \quad \Delta\phi_{\mathrm{GR}}.}$$

The planetary perturbations and other solar contributions should be treated separately rather than absorbed into the PPC correction.

Thus,

$$\boxed{\Delta\phi_{\mathrm{PPC}} = \Delta\phi_{\mathrm{GR}} + \delta\phi_{\mathrm{PPC}}.}$$

---

4. PPC Correction Parameter

For the PPC orbital equation, define

$$\boxed{\Delta\phi_{\mathrm{PPC}} = \Delta\phi_{\mathrm{GR}} \left(1 + \frac{\beta_{\mathrm{PPC}}}{3}\right).}$$

Therefore,

$$\boxed{\frac{\beta_{\mathrm{PPC}}}{3} = \frac{\Delta\phi_{\mathrm{PPC}}}{\Delta\phi_{\mathrm{GR}}} - 1.}$$

Hence,

$$\boxed{\beta_{\mathrm{PPC}} = 3\left(\frac{\Delta\phi_{\mathrm{PPC}}}{\Delta\phi_{\mathrm{GR}}} - 1\right).}$$

5. Case 2 PPC Numerical Result

For the proposed Case 2 value,

$$\boxed{\Delta\phi_{\mathrm{PPC}} = 42.9987''/\mathrm{century}}$$

and the reference value is

$$\boxed{\Delta\phi_{\mathrm{GR}} = 42.981''/\mathrm{century}.}$$

Therefore,

$$\beta_{\mathrm{PPC}} = 3\left(\frac{42.9987}{42.981} - 1\right).$$

Numerically,

$$\frac{42.9987}{42.981} - 1 \approx 4.35 \times 10^{-4}.$$

Thus,

$$\boxed{\beta_{\mathrm{PPC}} \approx 1.305 \times 10^{-3}.}$$

or

$$\boxed{\beta_{\mathrm{PPC}} \approx 0.1305\%.}$$

The corresponding difference from the GR reference is

$$\boxed{42.9987 - 42.981 = 0.0177''/\mathrm{century}.}$$

Thus the proposed PPC correction is very small compared with the overall Mercury perihelion motion.

6. Interpretation for PPC Case 2

Case 2 assumes a **very small but finite PPC source**:

$$\boxed{E_d > 0, \qquad E_d \text{ very small},}$$

$$\boxed{\omega > 0, \qquad \omega \text{ very small},}$$

and

$$\boxed{P_g = \omega E_d.}$$

Therefore,

$$\boxed{P_g \text{ is very small but not identically zero.}}$$

The PPC metric is then

$$\boxed{g_{\mu\nu}^{\mathrm{PPC}} = g_{\mu\nu}(E_d, P_g),}$$

and the PPC correction must ultimately follow from

$$\boxed{g_{\mu\nu}^{\mathrm{PPC}} \to \Gamma_{\alpha\beta}^{\mu} \to \mathrm{geodesic} \to \Delta\phi_{\mathrm{PPC}}.}$$

The proposed Case 2 result is therefore

$$\boxed{\Delta\phi_{\mathrm{PPC}} = 42.9987''/\mathrm{century}}$$

with

$$\boxed{\beta_{\mathrm{PPC}} \approx 1.305 \times 10^{-3}.}$$

7. Case 3 Is Different

Case 3 is the mathematical near-vacuum limit:

$$\boxed{E_d \to 0, \qquad P_g \to 0, \qquad |\nabla P_g| \to 0.}$$

Provided the PPC field equations have a regular continuous vacuum limit,

$$\boxed{g_{\mu\nu}^{\mathrm{PPC}} \to g_{\mu\nu}^{\mathrm{Schwarzschild}}.}$$

Then

$$\boxed{\beta_{\mathrm{PPC}} \to 0}$$

$$\boxed{g_{\mu\nu}^{\mathrm{PPC}} \to g_{\mu\nu}^{\mathrm{Schwarzschild}}.}$$

Then

$$\boxed{\beta_{\mathrm{PPC}} \to 0}$$

and

$$\boxed{\Delta\phi_{\mathrm{PPC}} \to 42.981''/\mathrm{century}.}$$

Thus Case 3 is a **limiting/recovery statement**, whereas Case 2 is intended to be a **finite, testable PPC prediction**.

8. Important Scientific Distinction

The parameter

$$\boxed{\beta_{\mathrm{PPC}} \approx 1.305 \times 10^{-3}}$$

should be described as a **PPC-specific orbital correction parameter**.

It should **not automatically be identified with the standard PPN parameter** $\beta - 1$. That identification would require deriving the correspondence between your PPC metric and the standard PPN metric.

Therefore the clean terminology is:

$$\boxed{\beta_{\mathrm{PPC}} = \mathrm{PPC\ orbital\ correction\ parameter.}}$$

9. Final Structure

The three cases can now be summarized as:

Case 1 — Exact Vacuum

$$\boxed{E_d = P_g = 0 \Rightarrow G_{\mu\nu} = 0 \Rightarrow \text{Schwarzschild} \Rightarrow 42.981''/\text{century}.}$$

Case 2 — Very Small Finite PPC Source

$$\boxed{E_d > 0, \quad \omega > 0, \quad E_d, \omega \ll 1}$$

$$\Downarrow$$

$$\boxed{\Delta\phi_{\text{PPC}} = 42.9987''/\text{century}}$$

$$\Downarrow$$

$$\boxed{\beta_{\text{PPC}} \approx 1.305 \times 10^{-3}.}$$

Case 3 — Continuous Near-Vacuum Limit

$$\boxed{E_d, P_g, \nabla P_g \to 0}$$

$$\Downarrow$$

$$\boxed{g_{\mu\nu}^{\mathrm{PPC}} \to g_{\mu\nu}^{\mathrm{Schwarzschild}}}$$

$$\Downarrow$$

$$\boxed{\beta_{\mathrm{PPC}} \to 0}$$

$$\Downarrow$$

$$\boxed{\Delta\phi_{\mathrm{PPC}} \to 42.981''/\mathrm{century}.}$$

This separation is important: Case 1 is the exact GR recovery, Case 2 is [illegible] proposed finite PPC prediction, and Case 3 is the mathematical continuity limit back to GR.

	Case 1	Case 2	Case 3
E_d	0	very small, finite	$\to 0$
P_g	0	very small, finite	$\to 0$
∇P_g	0	very small	$\to 0$
R	0	very small, generally nonzero	$\to 0$
$g_{\mu\nu}^{\mathrm{PPC}}$	$g_{\mu\nu}^{\mathrm{Schw.}}$	near-Schwarzschild	$\to g_{\mu\nu}^{\mathrm{Schw.}}$
β_{PPC}	0	$\approx 1.305 \times 10^{-3}$ (proposed)	$\to 0$
$\Delta\phi$	$42.981''$	$42.9987''$ (proposed)	$\to 42.981''$

DERIVATION OF THE THREE PPC CASES

1. COMMON PPC PERFECT-FLUID FRAMEWORK

The PPC energy-momentum tensor is

$T_{\mu\nu}^{PPC} = (E_d + P_g)u_\mu u_\nu + P_g g_{\mu\nu}$

with

$E_d = \rho c^2$

and

$P_g = \omega E_d$.

The PPC tensor field equation is

$G_{\mu\nu} = (8\pi G/c^4)\, T_{\mu\nu}^{PPC}$.

For the metric signature (−,+,+,+), the trace is

$T^{PPC} = -E_d + 3P_g$.

Therefore, the Ricci scalar is

$R = (8\pi G/c^4)(E_d - 3P_g)$.

Using $P_g = \omega E_d$,

$R = (8\pi G/c^4)\, E_d(1 - 3\omega)$.

The static weak-field PPC equation is

$\nabla^2 \Phi_{PPC} = (4\pi G/c^2)(E_d + 3P_g)$.

Therefore,

$\nabla^2 \Phi_{PPC} = 4\pi G\rho(1 + 3\omega)$.

The PPC metric is then determined from the PPC gravitational potential and the full PPC field equations:

$E_d, P_g \rightarrow \Phi_{PPC} \rightarrow g_{\mu\nu}^{PPC}$.

The inverse metric gives the connection,

$\Gamma^{\mu}_{\alpha\beta} = \tfrac{1}{2}\, g^{\mu\nu}(\partial_\alpha g_{\nu\beta} + \partial_\beta g_{\nu\alpha} - \partial_\nu g_{\alpha\beta})$.

The test-body motion is

$d^2x^\mu/d\tau^2 + \Gamma^{\mu}_{\alpha\beta}(dx^\alpha/d\tau)(dx^\beta/d\tau) = 0$.

For Mercury,

$u = 1/r$.

Define the PPC orbital correction by

$\Delta\varphi_{PPC} = \Delta\varphi_{GR}(1 + \beta_{PPC}/3)$.

CASE 1 — EXACT VACUUM / CLAIM A

The exact vacuum condition is

$E_d = 0$.

Since

$P_g = \omega E_d$,

we obtain

$P_g = 0$.

Therefore,

$T_{\mu\nu}^{PPC} = 0$

and

$G_{\mu\nu} = 0$.

The Ricci scalar becomes

$R = 0$.

For a static, spherically symmetric, asymptotically flat vacuum, the solution is the Schwarzschild metric.

Therefore,

$g_{\mu\nu}^{PPC} = g_{\mu\nu}^{Schwarzschild}$.

The Mercury orbital equation becomes

$u'' + u = GM_\odot/h^2 + (3GM_\odot/c^2)u^2$.

The perihelion advance is

$\Delta\varphi_{GR} = 6\pi GM_\odot / [a(1 - e^2)c^2]$.

For Mercury,

$\Delta\varphi_{GR} \approx 42.981''$/century.

Rounded,

$\Delta\varphi_{GR} \approx 43''$/century.

Thus,

$\beta_{PPC} = 0$

for the exact vacuum case.

CLAIM A

$E_d = P_g = 0$

↓

$G_{\mu\nu} = 0$

↓

Schwarzschild

↓

$\Delta\varphi_{PPC} \approx 42.981''$/century

Therefore:

PPC recovers the GR vacuum prediction for Mercury's perihelion advance.

CASE 2 — VERY SMALL BUT FINITE PPC SOURCE

Case 2 is different from the exact-vacuum case.

Here,

$E_d > 0$, but E_d is extremely small

and

$\omega > 0$, but ω is extremely small.

Therefore,

$P_g = \omega E_d$

is also extremely small but is not identically zero.

The static weak-field equation becomes

$\nabla^2\Phi_{PPC} = (4\pi G/c^2)E_d(1 + 3\omega)$.

The Ricci scalar is

$$R = (8\pi G/c^4)E_d(1 - 3\omega).$$

Thus, unlike Case 1, the PPC source is formally nonzero.

The PPC metric must therefore be calculated from

$$g_{\mu\nu}^{PPC} = g_{\mu\nu}(E_d, P_g).$$

The connection is then obtained from

$$\Gamma^{\mu}_{\alpha\beta}[g^{PPC}].$$

The orbital equation is parameterized as

$$u'' + u = GM_\odot/h^2 + (3 + \beta_{PPC})(GM_\odot/c^2)u^2.$$

The corresponding perihelion advance is

$$\Delta\varphi_{PPC} = \Delta\varphi_{GR}(1 + \beta_{PPC}/3).$$

For the proposed Case 2 numerical value,

$$\Delta\varphi_{PPC} = 42.9987''/\text{century}.$$

Using

$$\Delta\varphi_{GR} = 42.981''/\text{century},$$

we obtain

$$\beta_{PPC} = 3[(42.9987/42.981) - 1].$$

Therefore,

$$\beta_{PPC} \approx 0.001305.$$

Thus,

$$\beta_{PPC} \approx 1.305 \times 10^{-3}$$

or approximately

$$\beta_{PPC} \approx 0.1305\%.$$

CASE 2 RESULT

Very small finite E_d + very small ω

↓

$P_g = \omega E_d$

↓

PPC metric correction

↓

$\beta_{PPC} \approx 1.305 \times 10^{-3}$

↓

$\Delta\varphi_{PPC} = 42.9987''$/century

This is intended as the **independent PPC finite-source prediction**.

However, the value 42.9987″/century must ultimately be obtained from the explicit PPC metric and geodesic calculation rather than inserted as an assumption.

CASE 3 — CONTINUOUS NEAR-VACUUM LIMIT

Case 3 is a mathematical limiting case.

Take

$E_d \to 0$

and therefore

$P_g \to 0$.

Also require

$|\nabla P_g| \to 0$.

Then,

$E_d + 3P_g \to 0$.

Therefore,

$\nabla^2\Phi_{PPC} \to 0$.

The Ricci scalar also satisfies

$R \to 0$.

Provided the PPC field equations have a regular continuous vacuum limit,

$g_{\mu\nu}^{PPC} \to g_{\mu\nu}^{Schwarzschild}$.

Consequently,

$\Gamma^{\mu}_{\alpha\beta}{}^{PPC} \to \Gamma^{\mu}_{\alpha\beta}{}^{Schwarzschild}$.

The orbital correction therefore satisfies

$\beta_{PPC} \to 0$.

Hence,

$\Delta\varphi_{PPC} \to \Delta\varphi_{GR}$.

Therefore,

$\Delta\varphi_{PPC} \to 42.981''$/century.

CASE 3 RESULT

$E_d \to 0$

↓

$P_g \to 0$

↓

$\nabla P_g \to 0$

↓

$g_{\mu\nu}^{PPC} \rightarrow g_{\mu\nu}^{Schwarzschild}$

↓

$\beta_{PPC} \rightarrow 0$

↓

$\Delta\varphi_{PPC} \rightarrow 42.981''$/century

FINAL THREE-CASE STRUCTURE

CASE 1 — EXACT VACUUM

$E_d = P_g = 0$

→ Schwarzschild

→ $\beta_{PPC} = 0$

→ 42.981″/century

CASE 2 — VERY SMALL FINITE PPC SOURCE

$E_d > 0$, very small

$\omega > 0$, very small

→ $P_g = \omega E_d$

→ finite PPC correction

→ $\beta_{PPC} \approx 0.001305$

→ 42.9987″/century

CASE 3 — CONTINUOUS NEAR-VACUUM LIMIT

$E_d \to 0$

$P_g \to 0$

$\nabla P_g \to 0$

$\to g_{\mu\nu}^{PPC} \to$ Schwarzschild

$\to \beta_{PPC} \to 0$

$\to$ 42.981″/century

CENTRAL PPC LOGIC

$T_{\mu\nu}^{PPC}$

↓

E_d, P_g

↓

Φ_{PPC}

↓

$g_{\mu\nu}^{PPC}$

↓

$g^{\mu\nu}_{PPC}$

↓

$\Gamma^{\mu}_{\alpha\beta}$

↓

Mercury geodesic

↓

β_{PPC}

↓

Δφ_PPC

This separates the three claims cleanly:

Case 1 = GR recovery

Case 2 = finite PPC prediction

Case 3 = continuous PPC → GR limit.

1. Case 1 — Accurate

Case 1 is sound as a **vacuum/Schwarzschild recovery**:

E_d = P_g = 0

→ **G_μv = 0**

→ **Schwarzschild**

→ **42.98″/century**.

NASA describes the relativistic contribution to Mercury's perihelion as approximately **42.98″/century** and states that the observed residual is consistent with GR after the planetary perturbations are modeled.

Claim A = PPC recovery of the GR vacuum result.

---

2. Case 2 — Proposed 42.9987″ needs to be treated as a PPC prediction

Case 2 assumes:

E_d > 0, but very small

ω > 0, but very small

P_g = ωE_d

and proposes

Δφ_PPC = 42.9987″/century.

Numerically, relative to reference (42.981″),

β_PPC ≈ 0.001305

using definition

Δφ_PPC = Δφ_GR(1 + β_PPC/3).

That arithmetic is correct.

However, **NASA does not provide 42.9987″/century as an observed Mercury value**. NASA's material treats the relativistic contribution as approximately 42.98″/century and emphasizes that planetary perturbations and solar effects must be modeled separately.

So 42.9987″ should be labeled:

“PPC Case 2 theoretical prediction”

—not “NASA observed value.”

---

3. Case 3 — Limiting argument is appropriate

Case 3 says:

E_d → 0

P_g → 0

|∇P_g| → 0

→ g_μv^PPC → g_μv^Schwarzschild

→ β_PPC → 0

→ Δφ_PPC → 42.981″/century.

This is a **mathematical continuity hypothesis**, not an observational statement.

I would explicitly write:

Provided the PPC field equations possess a regular continuous vacuum limit.

That qualification is important because the continuity of the metric and of orbital coefficient has to follow from the actual PPC field equations.

---

4. One correction concerning NASA and the “deviation”

NASA's material actually supports decision to separate the effects.

NASA says that **planetary gravitational tugs account for most of Mercury's precession**, while the **second-largest contribution is the relativistic effect of the Sun**, with additional smaller effects from the Sun's interior structure, oblateness, and angular momentum.

An older NASA technical analysis explicitly gives the relativistic perihelion coefficient in terms of PPN parameters and notes the role of the solar quadrupole J_2.

Case 1

Exact vacuum

$E_d = P_g = 0$

→ $G_{\mu\nu} = 0$

→ Schwarzschild

→ $\Delta\varphi = 42.981''$/century ≈ 43″/century.

Case 2

Very small but finite PPC source

$E_d > 0$

$\omega > 0$

$E_d, \omega \ll 1$

→ $P_g = \omega E_d$

→ PPC metric correction

→ $\beta_{PPC} \approx 1.305 \times 10^{-3}$

→ proposed $\Delta\varphi_{PPC} = 42.9987''$/century.

Case 3

Continuous near-vacuum limit

$E_d \to 0$

$P_g \to 0$

$\nabla P_g \to 0$

→ $g_{\mu\nu}^{PPC} \to g_{\mu\nu}^{Schwarzschild}$

→ $\beta_{PPC} \to 0$

→ $\Delta\varphi_{PPC} \to 42.981''$/century.

Bottom line

Numerical comparison is internally consistent. NASA supports the approximately **43″/century relativistic benchmark and the need to separate planetary and solar contributions**.

But **NASA does not validate the specific PPC prediction 42.9987″/century**. That number has to come from PPC metric/geodesic derivation. That is exactly where Case 2 becomes the decisive test of theory.

Statement on the Three PPC Cases and Mercury Perihelion Advance

The PPC analysis distinguishes three physically different regimes.

Case 1 — Exact Vacuum:
When

$E_d = 0, \quad P_g = 0,$

the PPC field equations reduce to

$G_{\mu\nu} = 0,$

and the static, spherically symmetric exterior solution is Schwarzschild. The resulting relativistic contribution to Mercury's perihelion advance is

$$\Delta\varphi_{GR} \approx 42.981''/\text{century} \approx 43''/\text{century}.$$

This constitutes the **Claim A recovery of the GR vacuum prediction**.

Case 2 — Very Small but Finite PPC Source:
The PPC model instead considers

$$E_d > 0, \quad E_d \ll 1,$$

with

$$\omega > 0, \quad \omega \ll 1,$$

so that

$$P_g = \omega E_d$$

is very small but not identically zero. The resulting PPC metric is expected to differ only slightly from the Schwarzschild metric. For the proposed PPC value

$$\Delta\varphi_{PPC} = 42.9987''/\text{century},$$

and using

$$\Delta\varphi_{PPC} = \Delta\varphi_{GR}(1 + \beta_{PPC}/3),$$

with

$$\Delta\varphi_{GR} = 42.981''/\text{century},$$

one obtains

$$\beta_{PPC} \approx 1.305 \times 10^{-3}.$$

This represents the proposed **finite-source PPC correction** and must ultimately be derived from the explicit PPC metric and Mercury geodesic.

Case 3 — Continuous Near-Vacuum Limit:
When

$$E_d \to 0, \quad P_g \to 0, \quad |\nabla P_g| \to 0,$$

and provided the PPC field equations possess a regular continuous vacuum limit,

$g_{\mu\nu}^{PPC} \to g_{\mu\nu}^{Schwarzschild}$,

and therefore

$\beta_{PPC} \to 0$.

Consequently,

$\Delta\varphi_{PPC} \to 42.981''$/century.

Observational interpretation

NASA's treatment of Mercury distinguishes the large Newtonian planetary perturbations from the approximately **43″/century relativistic contribution**, while also accounting for smaller solar effects. Therefore, the PPC comparison should be made specifically with the **relativistic residual**, not with Mercury's total perihelion precession.

Accordingly, **42.981″/century is the GR reference value, approximately 43″/century is the observationally rounded relativistic value, and 42.9987″/century is a proposed PPC Case 2 prediction—not a NASA observational value.**

The decisive test of Case 2 remains:

$E_d, P_g \to g_{\mu\nu}^{PPC} \to \Gamma^{\mu}_{\alpha\beta} \to$ Mercury geodesic $\to \beta_{PPC} \to \Delta\varphi_{PPC}$.